Day 145 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{5}{x-1}$, $x \neq 1$.

Find $f^{-1}(x)$.

$y = \frac{5}{x-1}$, so $y(x-1) = 5$

$x - 1 = \frac{5}{y}$, so $x = \frac{5}{y} + 1$

Answer: $f^{-1}(x) = \frac{5}{x} + 1$

2 Exact trigonometric values (All boards)

Write down the exact value of tan 120°.

tan 120° has the same size as tan 60°, and the sign comes from the quadrant (CAST).

tan 60° = $\sqrt{3}$, so tan 120° = $-\sqrt{3}$

Answer: $-\sqrt{3}$

3 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (5, 4) and B is (15, −6).

Find (a) the midpoint of AB,

(b) the length of AB, as a surd in simplest form.

(a) Midpoint = $(\frac{5+15}{2}, \frac{4+(-6)}{2}) = (10, -1)$

(b) $AB^2 = (10)^2 + (-10)^2 = 100 + 100 = 200$

$AB = \sqrt{200} = 10\sqrt{2}$

Answer: (a) (10, −1) (b) $10\sqrt{2}$

4 Geometric distribution (CIE)

A fair die is rolled until a six appears. X is the number of rolls.

Find P(X = 6), to 3 s.f.

$P(X = 6) = (1 - p)^5 \times p = (\frac{5}{6})^5 \times \frac{1}{6}$

Answer: 0.0670

5 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos\alpha = \frac{12}{13}$. Find P and Q.

Horizontally: $P = 39\cos\alpha = 39 \times \frac{12}{13}$

Vertically: $Q = 39\sin\alpha = 39 \times \frac{5}{13}$

Answer: P = 36, Q = 15

6 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x-4)^2$, $x \geq 0$.

Find the range of f.

The minimum of $(x-4)^2$ is 0 at x = 4, which is allowed since $4 \geq 0$.

So $f(x) \geq 0$.

Answer: $f(x) \geq 0$

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7 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 10t + 21)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 10t + 21 = 0$
$(t - 3)(t - 7) = 0$

(b) $a = \frac{dv}{dt} = 2t - 10$; at $t = 3$, $a = 6 - 10$

Answer: (a) t = 3 and t = 7 (b) −4 m s^{-2}

8 Integrate using partial fractions (All boards)

$\frac{6x + 20}{(x + 2)(x + 4)} \equiv \frac{A}{x + 2} + \frac{B}{x + 4}$.

Find A and B, and hence find $\int \frac{6x + 20}{(x + 2)(x + 4)}\,dx$.

$6x + 20 \equiv A(x + 4) + B(x + 2)$
$x = -2$ gives $A = 4$; $x = -4$ gives $B = 2$
$\int \left(\frac{4}{x + 2} + \frac{2}{x + 4}\right) dx = 4 \ln|x + 2| + 2 \ln|x + 4| + c$

Answer: A = 4, B = 2; 4 ln |x + 2| + 2 ln |x + 4| + c

9 Normal approximation to the binomial (CIE)

X ~ B(200, 0.3). Use a normal approximation to find P(X ≤ 57), to 3 s.f.

$np = 60$ and $npq = 42$, so use Y ~ N(60, 42)
Continuity correction: P(Y < 57.5)
$z = (57.5 - 60) \div \sqrt{42}$

Answer: 0.350

10 Area between a line and a curve (All boards)

The curve $y = x^2$ and the line $y = 6x$ meet at the origin and at P.
Find the area enclosed between them.

$x^2 = 6x$ gives $x = 0$ or $x = 6$

Area $= \int_0^6 (6x - x^2)\,dx = \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 6

$= \frac{6^3}{2} - \frac{6^3}{3} = 36$

Answer: 36