Day 144 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 9 N and 12 N act on a boat at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 9 N force, to 1 d.p.

(a) $R^2 = 9^2 + 12^2 = 225$

$R = \sqrt{225}$

(b) $\tan\theta = \frac{12}{9}$

Answer: (a) 15 N (b) 53.1°

2 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with hours of revision from 4 to 14, the regression line is E = 13 + 6h.
Estimate exam score when h = 2 and comment on the reliability.

E = 13 + 6 × 2 = 25

2 is outside the range 4 to 14, so this is extrapolation.

Answer: E = 25; estimate is unreliable, as 2 is outside the data range (extrapolation)

3 Change of sign to locate a root (All boards)

$f(x) = x^3 - 2x - 5$.
Show that f(x) = 0 has a root between x = 2 and x = 3.

f(2) = −1.00
f(3) = 16.0
f is continuous and there is a change of sign, so a root lies in the interval

Answer: f(2) = −1.00, f(3) = 16.0: change of sign

4 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 4.

Centre (5, 4): $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 5)^2 + (y - 4)^2 = 16$

5 Addition rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.45, P(B) = 0.35 and P(A ∪ B) = 0.6.
(a) Find P(A ∩ B).
(b) Find P(A′ ∩ B′).

(a) P(A ∩ B) = P(A) + P(B) − P(A ∪ B) = 0.45 + 0.35 − 0.6 = 0.2
(b) A′ ∩ B′ is outside both, so P = 1 − P(A ∪ B) = 1 − 0.6

Answer: (a) 0.2 (b) 0.4

6 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{x - 15}{(x + 1)(x - 3)}$ in partial fractions.

Write $\frac{x - 15}{(x + 1)(x - 3)} = \frac{A}{(x + 1)} + \frac{B}{(x - 3)}$

$x - 15 = A(x - 3) + B(x + 1)$
$x = -1$: $-16 = A(-4)$ so $A = 4$
$x = 3$: $-12 = B(4)$ so $B = -3$

Answer: $\frac{4}{(x + 1)} + \frac{-3}{(x - 3)}$

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7 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(6\mathbf{i} + 5\mathbf{j})$ N and $(9\mathbf{i})$ N act on a particle of mass 5 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(6\mathbf{i} + 5\mathbf{j}) + (9\mathbf{i})$
= $(15\mathbf{i} + 5\mathbf{j})$ N

F = ma: $a = \frac{1}{5}(15\mathbf{i} + 5\mathbf{j})$

Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

8 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $3x^2 + y^2 = 13$.
Find dy/dx in terms of x and y, and hence the gradient at (2, 1).

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{3x}{y}$

At (2, 1): $dy/dx = -6$

Answer: $dy/dx = -\frac{3x}{y}$; gradient -6

9 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$2x - 3 > -1$ and $x^2 + 3x - 10 < 0$.

$2x - 3 > -1$ gives $x > 1$.

$x^2 + 3x - 10 < 0$: $(x + 5)(x - 2) < 0$, so $-5 < x < 2$.

Both are true where the two ranges overlap.

Answer: $1 < x < 2$

10 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient 3 and intercept 1.3. Find n and a, giving a to 3 significant figures.

$\log y = \log a + n \log x$

Gradient $n = 3$

$\log a = 1.3$ so $a = 10^{1.3} = 20.0$

Answer: $n = 3$, $a = 20.0$