Day 143 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 5.

Centre $(-2, 5)$: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x + 2)^2 + (y - 5)^2 = 25$

2 Natural logs and e^x (All boards)

Solve (a) $e^x = 7$ (b) $\ln x = 2.5$
Give answers to 3 significant figures.

(a) $x = \ln 7 = 1.95$

(b) $x = e^{2.5} = 12.2$

Answer: (a) x = 1.95 (b) x = 12.2

3 Geometric distribution (CIE)

A fair die is rolled until a six appears. X is the number of rolls.
Find E(X).

$E(X) = 1 \div p = 1 \div \frac{1}{6}$

Answer: 6

4 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$ to estimate $\frac{\sin 0.05}{\cos 0.05}$.

$$\frac{\sin\theta}{\cos\theta} \approx \frac{\theta}{1 - \frac{\theta^2}{2}}$$

$$= \frac{0.05}{1 - 0.00125} = \frac{0.05}{0.9988} = 0.0501$$

Answer: 0.0501

5 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 3.5 m s^{-1}.
At that instant car B starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $3.5\,t = \frac{1}{2} \times 1 \times t^2$

$t(3.5 - 0.5\,t) = 0$, so $t = 0$ or $t = 3.5 \div 0.5$

Answer: 7 s

6 Equation of a tangent or normal (All boards)

Find the equation of the normal to the curve
$y = 3x^3 + x^2 + 4$ at the point where $x = 2$.

dy/dx = 9x^2 + 2x, so the gradient of the curve at x = 2 is 40
At x = 2, y = 32

Gradient of normal $= -\frac{1}{40}$

$y - 32 = (-\frac{1}{40})(x - 2)$

Answer: $y = -\frac{1}{40}x + \frac{641}{20}$

7 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(n, p) has mean 32 and variance 6.4.
Find n and p.

np = 32 and npq = 6.4
q = 6.4 ÷ 32 = 0.2, so p = 0.8
n = 32 ÷ 0.8
Answer: n = 40, p = 0.8

8 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$2x + 5 < 11$ and $x^2 - 7x + 6 < 0$.

$2x + 5 < 11$ gives $x < 3$.
$x^2 - 7x + 6 < 0$: $(x - 1)(x - 6) < 0$, so $1 < x < 6$.
Both are true where the two ranges overlap.
Answer: $1 < x < 3$

9 Friction: acceleration along rough ground (All boards)

A block of mass 10 kg is pulled along rough horizontal ground by a horizontal force of 54.4 N.
The coefficient of friction is 0.3. Take g = 9.8 m s^{-2}. Find the acceleration.

R = 10 × 9.8 = 98
F = μR = 0.3 × 98 = 29.4
P − F = ma: 54.4 − 29.4 = 10 a
Answer: 2.5 m s^{-2}

10 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 5x^2 - 16x + 80$
(a) Show that $(x - 4)$ is a factor of f(x).
(b) Factorise f(x) completely.

f(4) = 64 + (−80) + (−64) + 80 = 0, so $(x - 4)$ is a factor.
$f(x) = (x - 4)(x^2 - x - 20)$
$x^2 - x - 20 = (x + 4)(x - 5)$
Answer: (b) $f(x) = (x + 4)(x - 4)(x - 5)$