**Day 142** **A Level Maths: C Tier 10-a-day: Model Solutions**

## 1 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = (3x + 2)^3$
at the point where $x = 1$.

Use the chain rule: $dy/dx = 3 \times 3(3x + 2)^2$
At $x = 1$: $dy/dx = 225$
**Answer: 225**

## 2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(2x)^3 \div 2x^2$.

$(2x)^3 = 8x^3$
$8x^3 \div 2x^2$: divide the numbers and subtract the powers
$= 4x$
**Answer: 4x**

## 3 Stationary points of a cubic (All boards)

$y = x^3 + 3x^2 - 9x + 5$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 + 6x - 9 = 3(x + 3)(x - 1) = 0$
$x = -3$: $y = 32$; $x = 1$: $y = 0$
$d^2y/dx^2 = 6x + 6$: at $x = -3$ it is $-12$ (maximum), at $x = 1$ it is 12 (minimum)
**Answer: (−3, 32) maximum; (1, 0) minimum**

## 4 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(12, p)$. $H_0$: $p = 0.25$, $H_1$: $p < 0.25$.
In a sample, $x = 0$. It is given that $P(X \leq 0) = 0.0317$.
Test $H_0$ at the 5% significance level.

Compare 0.0317 with 0.05
$0.0317 < 0.05$, so reject $H_0$
**Answer: Reject $H_0$: evidence that p is less than 0.25**

## 5 Discriminant: equal roots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x^2 + kx + 4 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$
$k^2 - 4 \times 1 \times 4 = 0$, so $k^2 = 16$
**Answer: k = ±4**

## 6 Integration by substitution (All boards)

Use the substitution $u = \sin x$ to find
$\int 3 \cos x \sin^4 x \, dx$.

$u = \sin x$, so $du = \cos x \, dx$
$\int 3u^4 \, du = \frac{3u^5}{5}$
Replace u by sin x
**Answer: $\frac{3 \sin^5 x}{5} + c$**

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## 7 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 2 m $s^{-1}$ to 10 m $s^{-1}$ as it travels 96 m. Find the acceleration.

$v^2 = u^2 + 2as$

$10^2 = 2^2 + 2 \times a \times 96$

$96 = 192\ a$

**Answer: 0.5 m $s^{-2}$**

## 8 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + \frac{1}{2}x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in $x^2$. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{3}$, $y = \frac{1}{2}x$

$= 1 + \frac{1}{3}(\frac{1}{2}x) + \frac{\frac{1}{3}((-\frac{2}{3}))}{2}(\frac{1}{2}x)^2$

$= 1 + \frac{1}{6}x - \frac{1}{36}x^2$

Valid when $|\frac{1}{2}x| < 1$, i.e. $|x| < 2$

**Answer: $1 + \frac{1}{6}x - \frac{1}{36}x^2$ ; $|x| < 2$**

## 9 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

For data set A, r = 0.83. For data set B, r = 0.32.
Which data set has the stronger linear correlation? Give a reason.

Strength depends on the size of r, ignoring the sign.

|0.83| = 0.83 and |0.32| = 0.32.

**Answer: data set A, as |r| is larger**

## 10 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist starts from rest and accelerates uniformly at 2 m $s^{-2}$ for 10 s, then travels at constant speed for a further 15 s. Find the total distance travelled.

Speed reached: $v = 2 \times 10 = 20$ m $s^{-1}$

Stage 1: $s = \frac{1}{2} \times 20 \times 10 = 100$

Stage 2: $s = 20 \times 15 = 300$

**Answer: 400 m**