Day 140 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Integrate (ax + b)^n and related (All boards)

Find $\int (4x + 1)^3 \, dx$.

Reverse the chain rule: raise the power by 1, divide by the new power and by 4
Here a = 4

Answer: $\frac{(4x + 1)^4}{16} + c$

2 Mean from a frequency table (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises the number of pets owned by 50 people.
Pets, x: 1, 2, 3, 4, 5
Frequency, f: 9, 17, 13, 3, 8
Find the mean.

$\Sigma fx = 9\times1 + 17\times2 + 13\times3 + 3\times4 + 8\times5 = 134$
Mean = $\Sigma fx \div \Sigma f = 134 \div 50$
Answer: 2.68

3 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−4, −2) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(x) + 5 (b) y = f(4x)

(a) y = f(x) + 5 adds 5 to y, so P maps to (−4, 3).
(b) y = f(4x) divides x by 4, so P maps to (−1, −2).
Answer: (a) (−4, 3) (b) (−1, −2)

4 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arccos(\frac{1}{2})$ (b) arcsin(1)

(a) y = $\arccos(\frac{1}{2})$ means $\cos y = \frac{1}{2}$ with $0 \le y \le \pi$: $y = \frac{\pi}{3}$

(b) y = arcsin(1) means $\sin y = 1$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$: $y = \frac{\pi}{2}$

Answer: (a) $\frac{\pi}{3}$ **(b)** $\frac{\pi}{2}$

5 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 24 terms of the series
−5 − 3 − 1 + 1 + ...

a = −5, d = 2, n = 24
$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{24}{2}[2(-5) + 23 \times 2]$
= 432
Answer: 432

6 Quadratic in sin or cos (All boards)

Solve $\cos^2 x + \cos x = 0$ for $0° \le x \le 360°$.

Factorise: cos x(cos x + 1) = 0 (as a quadratic in cos x)
cos x = 0 or cos x = −1
x = 90°, 180°, 270°
Answer: x = 90°, 180°, 270°

7 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a geometric sequence $u_2 = -4$ and $u_3 = 8$.

Find the first term and the common ratio.

$r = u_3 \div u_2 = 8 \div (-4) = -2$

$a = u_2 \div r^1 = 2$

Answer: a = 2, r = −2

8 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 30 kg and length 8 m rests horizontally on supports at A and B. A load of mass 20 kg is placed on the beam 6 m from A. Take $g = 9.8$ m s^{-2}. Find the reaction at B.

Moments about A:

$R_B \times 8 = 30 \times 9.8 \times 4 + 20 \times 9.8 \times 6$

$R_B \times 8 = 2352$

Answer: 294 N

9 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 10^2)$. H_0: $\mu = 40$, H_1: $\mu \neq 40$.

A sample of 36 has mean $\bar{x} = 44.17$.

Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (44.17 - 40) \div (10 \div 6)$

$z = 2.50$

2.50 is inside the critical region, so reject H_0

Answer: z = 2.50; reject H_0

10 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from rest from the top of a cliff of height 4.9 m. Take $g = 9.8$ m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $4.9 = \frac{1}{2} \times 9.8 \times t^2$

$4.9 = 4.9t^2$, so $t^2 = 1$

(b) $v = u + at = 9.8 \times 1$

Answer: (a) 1 s (b) 9.8 m s^{-1}