Day 14 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$.

Write down a vector equation of the line through the origin, parallel to this line.

Parallel lines have the same direction vector

The line passes through O, so the position vector is the zero vector

Answer: $\mathbf{r} = s\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

2 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 24 m s^{-1} brakes uniformly and comes to rest after travelling 96 m.

(a) Find the deceleration.

(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 24^2 + 2 \times a \times 96$

so $a = -3$, a deceleration of 3

(b) $s = \frac{(u + v)}{2}t$: $96 = 12t$

Answer: (a) 3 m s^{-2} (b) 8 s

3 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A) = 0.1, P(B) = 0.4 and P(A ∩ B) = 0.04.

(a) Find P(A | B).

(b) Are A and B independent? Give a reason.

(a) P(A | B) = P(A ∩ B) ÷ P(B) = 0.04 ÷ 0.4 = 0.1

(b) P(A) × P(B) = 0.1 × 0.4 = 0.04

This equals P(A ∩ B) = 0.04, so A and B are independent

Answer: (a) 0.1 (b) independent, as P(A) × P(B) = P(A ∩ B)

4 Friction: constant speed, find the coefficient (All boards)

A crate of mass 20 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 58.8 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the crate and ground.

R = 20 × 9.8 = 196

Constant speed: F = P = 58.8

F = μR: 58.8 = μ × 196

Answer: 0.3

5 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.

Use the small angle approximations to find an approximation for 2 sin θ + cos θ.

$\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$

So $2\sin\theta + \cos\theta \approx 1 + 2\theta - \frac{\theta^2}{2}$

Answer: $\mathbf{1 + 2\theta - \frac{\theta^2}{2}}$

6 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that 3x + 9y = 91.

Assume that integers x and y exist with 3x + 9y = 91.

Then 3(x + 3y) = 91, so 91 would be a multiple of 3.

But 91 = 3 × 30 + 1, so it is not a multiple of 3.

This is a contradiction, so no such integers exist.

Answer: Proved

7 Addition formulae: exact values (All boards)

By writing 75° as 30° + 45°, find the exact value of cos 75°.

$\cos(30° + 45°) = \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$

Simplify: $\frac{\sqrt{6} - \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} - \sqrt{2}}{4}$

8 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are x = 2 and y = 1, so a = 2 and c = 1.

Through (0, −1): $-1 = \frac{k}{0 - 2} + 1$

$\frac{k}{-2} = -2$, so k = 4

Answer: a = 2, c = 1, k = 4

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.2, H_1: p > 0.2.

Find the critical region for a test at the 1% significance level.

$P(X \geq 8) = 0.00424 \leq 0.01$

$P(X \geq 7) = 0.0181 > 0.01$

Answer: X ≥ 8

10 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 + 8x - 1$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of $y = x^2 + 8x - 1$.

$x^2 + 8x = (x + 4)^2 - 16$

$x^2 + 8x - 1 = (x + 4)^2 - 17$

The minimum is where $(x + 4)^2 = 0$, i.e. x = −4

Answer: $(x + 4)^2 - 17$; minimum (−4, −17)