Day 139 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Driving force and resistance (All boards)

A car of mass 500 kg is driven along a straight horizontal road by a driving force of 1350 N against a constant resistance of 600 N.

Find the acceleration of the car.

Resultant force = 1350 − 600 = 750 N

F = ma: 750 = 500 a

Answer: 1.5 m s^{-2}

2 Double angle: find sin 2A and cos 2A (All boards)

A is acute and cos A = $\frac{4}{5}$.

Find the exact values of sin 2A and cos 2A.

sin A = $\sqrt{1 - \cos^2 A} = \frac{3}{5}$

sin 2A = 2 sin A cos A = $2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

cos 2A = 2cos^{2}A − 1 = $2 \times \frac{16}{25} - 1 = \frac{7}{25}$

Answer: sin 2A = $\frac{24}{25}$, cos 2A = $\frac{7}{25}$

3 Median and interquartile range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 7 values: 32, 6, 30, 79, 52, 42, 59

Find the median and the interquartile range.

Ordered: 6, 30, 32, 42, 52, 59, 79

Median = 42, lower quartile = 30, upper quartile = 59

IQR = 59 − 30

Answer: median = 42, IQR = 29

4 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + x - 12 \leq 0$.

Solve $x^2 + x - 12 = 0$: $(x + 4)(x - 3) = 0$

Critical values $x = -4$ and $x = 3$.

The graph is a ∪-shaped parabola, so take the region between the roots.

Answer: $-4 \leq x \leq 3$

5 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of X ~ N(60, 5^2).

a and b are equal distances from the mean. The shaded area, P(X > b), is 0.24.

Find P(a < X < b).

By symmetry P(X < a) = 0.24

P(a < X < b) = 1 − 2 × 0.24

Answer: 0.52

6 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -2$ and $y = -1$, so $a = -2$ and $c = -1$.

Through (0, 1): $1 = \frac{k}{0 - (-2)} + -1$

$\frac{k}{2} = 2$, so $k = 4$

Answer: $a = -2$, $c = -1$, $k = 4$

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7 Car towing a caravan (All boards)

A car of mass 1200 kg tows a caravan of mass 600 kg on a level road with acceleration 0.5 m s^{-2}. Resistances: 400 N on the car, 150 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 400 - 150 = 1800 \times 0.5$

Caravan: $T - 150 = 600 \times 0.5$

Answer: D = 1450 N, T = 450 N

8 Maximum value from R form (All boards)

$f(x) = 12 \sin x + 9 \cos x + 6$.

Write f(x) in the form $R \sin(x + \alpha) + 6$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{12^2 + 9^2} = 15$, $\tan \alpha = \frac{9}{12}$ so $\alpha = 36.9°$

$f(x) = 15 \sin(x + 36.9°) + 6$

$\sin(x + \alpha)$ has maximum value 1, so f(max) = 15 + 6

Answer: f(x) = 15 sin(x + 36.9°) + 6; maximum 21

9 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -1$, 1 and 3.

$f(x) = k(x + 1)(x - 1)(x - 3)$

At (0, −6): $k \times 1 \times (-1) \times (-3) = -6$

$k = -2$

Answer: f(x) = −2(x + 1)(x − 1)(x − 3)

10 Integration by substitution (All boards)

Use the substitution u = sin x to find

$\int 3 \cos x \sin^3 x \, dx$.

$u = \sin x$, so $du = \cos x \, dx$

$\int 3u^3 \, du = \frac{3u^4}{4}$

Replace u by sin x

Answer: $\frac{3 \sin^4 x}{4} + c$