Day 138 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{6} \times \sqrt{4}$, giving your answer in the form $p\sqrt{q}$, or as an integer.

$\sqrt{6} \times \sqrt{4} = \sqrt{24}$
$= 2\sqrt{6}$
Answer: $2\sqrt{6}$

2 Integrate standard functions (All boards)

Find
(a) $\int 12 \sec^2 4x \, dx$
(b) $\int 12 \cos 4x \, dx$

Use the standard integrals and divide by the coefficient of x where needed.
(a) $= 3 \tan 4x + c$
(b) $= 3 \sin 4x + c$
Answer: (a) $3 \tan 4x + c$ (b) $3 \sin 4x + c$

3 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 5)(x - 1)(x - 5)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -5, 1, 5$
$x = 0$: $y = 5 \times (-1) \times (-5) = 25$
Answer: $(-5, 0)$, $(1, 0)$, $(5, 0)$, $(0, 25)$

4 Solve a logarithmic equation (one log) (All boards)

Solve $\log_5 x = 4$.

$x = 5^4$
Answer: $x = 625$

5 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos \alpha = \frac{12}{13}$. Find P and Q.

Horizontally: $P = 65 \cos \alpha = 65 \times \frac{12}{13}$

Vertically: $Q = 65 \sin \alpha = 65 \times \frac{5}{13}$

Answer: $P = 60$, $Q = 25$

6 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of $X \sim N(115, 5^2)$.
a and b are equal distances from the mean. The shaded area is 0.77.
Find $P(X > b)$.

Unshaded area $= 1 - 0.77 = 0.23$
The two tails are equal by symmetry, so $P(X > b) = 0.23 \div 2$
Answer: 0.115

7 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(7\mathbf{i} + 5\mathbf{j})$ N and $(2\mathbf{i} - 2\mathbf{j})$ N act on a particle of mass 3 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant $= (7\mathbf{i} + 5\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j})$
$= (9\mathbf{i} + 3\mathbf{j})$ N

F = ma: $a = \frac{1}{3}(9\mathbf{i} + 3\mathbf{j})$

Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

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8 Quotient rule (All boards)

Use the quotient rule to find the derivative of

$y = \frac{\ln x}{x}$.

Quotient rule: if $y = \frac{u}{v}$ then $dy/dx = \frac{u'v - uv'}{v^2}$

Differentiate the numerator and denominator, then substitute and simplify.

Answer: $\mathbf{dy/dx = \frac{1 - \ln x}{x^2}}$

9 Normal approximation to the binomial (CIE)

X ~ B(120, 0.3). Use a normal approximation to find P(X ≥ 41), to 3 s.f.

np = 36 and npq = 25.2, so use Y ~ N(36, 25.2)

Continuity correction: P(Y > 40.5)

$z = (40.5 - 36) \div \sqrt{25.2}$

Answer: 0.185

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x + k$ is a tangent to the curve

$y = x^2 - 8x - 4$.

Find the value of k.

$x^2 - 8x - 4 = -2x + k$

$x^2 - 6x + (-4 - k) = 0$

A tangent touches once: discriminant = 0

$(-6)^2 - 4(-4 - k) = 0$

Answer: $\mathbf{k = -13}$