Day 137 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of y = f(x) + 5.

y = f(x) + 5 is obtained from y = f(x) by a:

translation by the vector $\begin{pmatrix}0\\5\end{pmatrix}$

Answer: translation by the vector $\begin{pmatrix}\mathbf{0}\\\mathbf{5}\end{pmatrix}$

2 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of y = a sin bx for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 4, so a = 4

The graph completes 2 full cycles in 360°, so the period is 180° and $b = \frac{360}{180} = 2$

Answer: a = 4, b = 2

3 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 + 2t^2 - t$.
Find its velocity when t = 4.

$v = \frac{ds}{dt} = 6t^2 + 4t - 1$

When t = 4: $v = 6 \times 4^2 + 4 \times 4 - 1$

Answer: 111 m s^{-1}

4 Double angle equation (All boards)

Solve $\cos 2x = \frac{1}{2}$ for $0° \leq x \leq 180°$.

Let u = 2x, so $0° \leq u \leq 360°$

$\cos u = \frac{1}{2}$: u = 60°, 300°, 420°, 660°

Divide by 2: x = 30°, 150°

Answer: x = 30°, 150°

5 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 4$, $x \geq 0$.

Find $f^{-1}(x)$ and state its domain.

$y = x^2 - 4$, so $x^2 = y + 4$

$x \geq 0$, so $x = +\sqrt{y + 4}$

The domain of f^{-1} is the range of f: $y \geq -4$

Answer: $f^{-1}(x) = \sqrt{x + 4}$, $x \geq -4$

6 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \frac{5}{x_n^{\,2} + 1} + 1$, with $x_0 = 2$.

Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.0000$

$x_2 = 2.0000$

$x_3 = 2.0000$

Answer: $x_1 = 2.000$, $x_2 = 2.000$, $x_3 = 2.000$

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7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 1$ and $y = -2$, so $a = 1$ and $c = -2$.

Through $(0, -4)$: $-4 = \frac{k}{0 - 1} + -2$

$\frac{k}{-1} = -2$, so $k = 2$

Answer: a = 1, c = −2, k = 2

8 Box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plot summarises the journey times, in minutes, of some commuters.
Find the range and the interquartile range.

Range = 80 − 15 = 65
IQR = 55 − 25 = 30
Answer: range = 65, IQR = 30

9 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 10 m s^{-1}.

At that instant car B starts from rest at O with constant acceleration 4 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $10\,t = \frac{1}{2} \times 4 \times t^2$

$t(10 - 2\,t) = 0$, so $t = 0$ or $t = 10 \div 2$

Answer: 5 s

10 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 6^2)$. H_0: $\mu = 30$, H_1: $\mu \neq 30$.

A sample of 16 has mean $\bar{x} = 27.6$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (27.6 - 30) \div (6 \div 4)$
$z = -1.60$
-1.60 is outside the critical region, so do not reject H_0
Answer: z = −1.60; do not reject H_0