Day 135 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2(x + 5)(x + 2)(x - 2)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -5, -2, 2$
$x = 0$: $y = 5 \times 2 \times (-2) \times 2 = -40$
Answer: $(-5, 0)$, $(-2, 0)$, $(2, 0)$, $(0, -40)$

2 Integrate standard functions (All boards)

Find

(a) $\int \frac{3}{x}\, dx$

(b) $\int 12 \sin 4x\, dx$

Use the standard integrals and divide by the coefficient of x where needed.
(a) $= 3 \ln |x| + c$
(b) $= -3 \cos 4x + c$
Answer: (a) $3 \ln |x| + c$ (b) $-3 \cos 4x + c$

3 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(3\mathbf{i} - 5\mathbf{j})$ m s^{-1} and constant acceleration $(3\mathbf{i} - \mathbf{j})$ m s^{-2}.
Find its velocity after 3 s.

$v = u + at$
$v = (3\mathbf{i} - 5\mathbf{j}) + 3(3\mathbf{i} - \mathbf{j})$
$= (3\mathbf{i} - 5\mathbf{j}) + (9\mathbf{i} - 3\mathbf{j})$
Answer: $(12\mathbf{i} - 8\mathbf{j})$ m s^{-1}

4 Poisson distribution (CIE)

Emails arrive at an average rate of 2 per hour, at random.
Find the probability of at most 6 in 3 hours, to 3 s.f.

Mean for 3 hours $= 3 \times 2 = 6$
$P(X \leq 6) = P(0) + P(1) + ... + P(6)$, by calculator
Answer: 0.606

5 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a cyclist.
(a) Find the acceleration in the first 2 s.
(b) Find the deceleration in the last 6 s.
(c) Find the total distance travelled.

(a) $a = \frac{12 - 4}{2}$

(b) $a = \frac{12 - 0}{6}$, a deceleration of 2

(c) area $= \frac{1}{2}(4 + 12) \times 2 + 12 \times 12 + \frac{1}{2} \times 6 \times 12$

Answer: (a) 4 m s^{-2} (b) 2 m s^{-2} (c) 196 m

6 Product rule (All boards)

Use the product rule to differentiate
$y = x^3 e^{2x}$.

$u = x^3$, $v = e^{2x}$
$u' = 3x^2$, $v' = 2\, e^{2x}$
$dy/dx = u'v + uv'$, which simplifies to the answer below
Answer: $dy/dx = x^2 e^{2x}(2x + 3)$

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7 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 19x - 30$
(a) Show that $(x + 2)$ is a factor of $f(x)$.
(b) Factorise $f(x)$ completely.

$f(-2) = (-8) + 0 + 38 + (-30) = 0$, so $(x + 2)$ is a factor.
$f(x) = (x + 2)(x^2 - 2x - 15)$
$x^2 - 2x - 15 = (x + 3)(x - 5)$
Answer: (b) $f(x) = (x + 3)(x + 2)(x - 5)$

8 Normal approximation to the binomial (CIE)

$X \sim B(60, 0.3)$. Use a normal approximation to find $P(X \geq 18)$, to 3 s.f.

$np = 18$ and $npq = 12.6$, so use $Y \sim N(18, 12.6)$
Continuity correction: $P(Y > 17.5)$
$z = (17.5 - 18) \div \sqrt{12.6}$
Answer: 0.556

9 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins $-6, -7, -8, -9, ...$
Find (a) an expression for the nth term,
(b) the 11th term.

$a = -6$, $d = -1$
$u_n = a + (n - 1)d = -6 + (n - 1)(-1) = -n - 5$
$u_{11} = -16$
Answer: (a) $-n - 5$ (b) -16

10 Integrate using partial fractions (All boards)

$\frac{-4x - 15}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$.

Find A and B, and hence find $\int \frac{-4x - 15}{(x + 3)(x + 4)}\, dx$.

$-4x - 15 \equiv A(x + 4) + B(x + 3)$
$x = -3$ gives $A = -3$; $x = -4$ gives $B = -1$
$\int \left(\frac{-3}{x + 3} + \frac{-1}{x + 4}\right) dx = -3 \ln|x + 3| + -1 \ln|x + 4| + c$
Answer: $A = -3$, $B = -1$; $-3 \ln|x + 3| - \ln|x + 4| + c$