Day 134 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Degrees and radians (All boards)

(a) Convert $\frac{\pi}{6}$ radians to degrees.

(b) Convert 40° to radians, giving your answer in terms of π.

(a) $\frac{\pi}{6} \times \frac{180}{\pi} = 30°$

(b) $40 \times \frac{\pi}{180} = \frac{2\pi}{9}$

Answer: (a) 30° (b) $\frac{2\pi}{9}$ radians

2 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A company says that the probability a customer buys a ticket is 0.5.
A researcher believes that the probability is higher. Let p be the probability that customer buys a ticket.
Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.

One-tailed test

Answer: H_0: $p = 0.5$, H_1: $p > 0.5$

3 Parallel vectors (All boards)

$\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -15 \\ -3 \end{pmatrix}$.

Show that a and b are parallel and find the ratio |b| : |a|.

$\mathbf{b} = -3 \times \mathbf{a}$, since $-3 \times 5 = -15$ and $-3 \times 1 = -3$

So a and b are parallel and |b| : |a| = 3 : 1

Answer: $\mathbf{b} = -3\mathbf{a}$, so parallel; ratio 3 : 1

4 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 - 2x)^6$.

$(1 + y)^6 = 1 + 6y + 15y^2 + 20y^3 + \ldots$ with $y = -2x$

$= 1 + 6(-2x) + 15(-2x)^2 + 20(-2x)^3$

Answer: $1 - 12x + 60x^2 - 160x^3$

5 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 6 values: 10, 19, 11, 12, 2, 3

Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 57$, $\Sigma x^2 = 739$, $n = 6$

Mean $= 57 \div 6 = 9.500$

Variance $= \frac{739}{6} - (9.500)^2 = 32.92$

Standard deviation $= \sqrt{32.92}$

Answer: mean = 9.50, standard deviation = 5.74

6 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = **a**, OB = **b**, and P lies on AB with AP : PB = 3 : 2.

Find AB and OP in terms of **a** and **b**.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \mathbf{b} - \mathbf{a}$

$\mathrm{AP} = \frac{3}{5}\mathrm{AB} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$\mathrm{OP} = \mathrm{OA} + \mathrm{AP} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

Answer: $\mathbf{AB} = \mathbf{b} - \mathbf{a}$; $\mathbf{OP} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

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7 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 6x < 0$.

Solve $x^2 - 6x = 0$: $(x + 0)(x - 6) = 0$
Critical values $x = 0$ and $x = 6$.
The graph is a ∪-shaped parabola, so take the region between the roots.
Answer: $0 < x < 6$

8 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 8.25 m s^{-1}.
At that instant car Q starts from rest at O with constant acceleration 1.5 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $8.25\,t = \frac{1}{2} \times 1.5 \times t^2$
$t(8.25 - 0.75\,t) = 0$, so $t = 0$ or $t = 8.25 \div 0.75$
Answer: 11 s

9 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{-4}{x + 3} + 4$.
Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = -3$ and $y = 4$.
$y = 0$: $\frac{-4}{x + 3} = -4$, so $x = -2$
$x = 0$: $y = \frac{-4}{3} + 4 = \frac{8}{3}$
Answer: $x = -3$, $y = 4$; $(-2, 0)$, $(0, \frac{8}{3})$

10 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 144 m apart, and cycle directly towards each other at constant speeds of 6 m s^{-1} and 10 m s^{-1}. Find the time until they meet.

Closing speed = $6 + 10 = 16$ m s^{-1}
Time = $\frac{144}{16}$
Answer: 9 s