Day 131 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Laws of logarithms: single log (All boards)

Write 3 log 2 + log 5 − log 10 as a single logarithm, giving it in the form log N where N is an integer.

$3 \log 2 = \log 2^3 = \log 8$

$\log 8 + \log 5 - \log 10 = \log \frac{8 \times 5}{10} = \log 4$

Answer: log 4

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of size 21 is taken from 189 members: 108 Under 30, 81 30 to 50. How many 30 to 50 should be in the sample?

$\text{Number} = 21 \times \frac{81}{189}$

$= 9$

Answer: 9

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(2x)^2 \div 4x$.

$(2x)^2 = 4x^2$

$4x^2 \div 4x$: divide the numbers and subtract the powers

$= x$

Answer: x

4 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = 3x(2 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where x = 0 and x = 2

$\text{Area} = \int_0^2 3(2x - x^2)\,dx = [3(\frac{2x^2}{2} - \frac{x^3}{3})]$ from 0 to 2

$= 3(\frac{2^3}{2} - \frac{2^3}{3})$

$= 4$

Answer: 4

5 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(40, p) and E(X) = 4.
Find p and Var(X).

$np = 4$, so $p = 4 \div 40 = 0.1$

$\text{Var}(X) = npq = 4 \times 0.9$

Answer: p = 0.1, Var(X) = 3.6

6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 6| \geq 5$.

$x - 6 \geq 5$ or $x - 6 \leq -5$

Add 6 to each side.

Answer: $x \leq 1$ or $x \geq 11$

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7 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = 2t^2 - 12t$.
(a) Find its acceleration when t = 3.
(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 4t - 12$

(a) $t = 3$: $a = 4 \times 3 - 12$

(b) $4t - 12 = 0$, so $t = \frac{12}{4}$

Answer: (a) 0 m s^{-2} (b) 3 s

8 Product rule (All boards)

Use the product rule to differentiate
$y = e^{2x} \sin 4x$.

$u = e^{2x}$, $v = \sin 4x$

$u' = 2e^{2x}$, $v' = 4 \cos 4x$

$dy/dx = u'v + uv'$, which simplifies to the answer below

Answer: $dy/dx = e^{2x}(2 \sin 4x + 4 \cos 4x)$

9 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{4x + 5}{(x + 2)^2}$ in the form $\frac{A}{(x + 2)} + \frac{B}{(x + 2)^2}$.

$4x + 5 = A(x + 2) + B$

Coefficient of x: $A = 4$

Constant: $5 = -8 + B$, so $B = -3$

Answer: $A = 4$, $B = -3$

10 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points X and Y, 66 m apart, and cycle directly towards each other at constant speeds of 12 m s^{-1} and 10 m s^{-1}. Find the time until they meet.

Closing speed = 12 + 10 = 22 m s^{-1}

Time = $\frac{66}{22}$

Answer: 3 s