Day 130 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $7 \leq 2x + 5 \leq 13$.

Subtract 5 from each part: $2 \leq 2x \leq 8$
Divide each part by 2.
Answer: $1 \leq x \leq 4$

2 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(70, 5^2)$. Find $P(X < 80)$, giving your answer to 3 s.f.

Standardise: $z = (80 - 70) \div 5 = 2$
$P(Z < 2)$ from calculator
Answer: 0.977

3 3D vectors: distance, magnitude and midpoint (All boards)

A has coordinates $(4, -1, -3)$ and B has coordinates $(-6, 10, -1)$.
Find (a) the vector AB, (b) the distance AB, (c) the midpoint of AB.

(a) $\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} -10 \\ 11 \\ 2 \end{pmatrix}$

(b) $|\text{AB}| = \sqrt{10^2 + 11^2 + 2^2} = 15$

(c) Midpoint $= (-1, \frac{9}{2}, -2)$

Answer: (a) $\begin{pmatrix} -10 \\ 11 \\ 2 \end{pmatrix}$ (b) 15 (c) $(-1, \frac{9}{2}, -2)$

4 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 2 and common difference 2. Find the smallest n for which the sum of the first n terms exceeds 600.

$S_n = \frac{n}{2}[4 + 2(n - 1)] = \frac{n(2n + 2)}{2}$

$S_{24} = 600$ and $S_{25} = 650$

The sum first exceeds 600 when $n = 25$.
Answer: $n = 25$

5 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(5\mathbf{i} + 19.6\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take $g = 9.8$ m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$

$t = \frac{19.6}{4.9} = 4$

(b) $0 = 19.6^2 - 2 \times 9.8 \times H$

$H = \frac{384.16}{19.6}$

Answer: (a) 4 s (b) 19.6 m

6 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations
$x = 4t - 5$, $y = 3t^2 + 1$.
Find the coordinates of the point where $t = -2$.

$x = 4 \times (-2) - 5 = -13$

$y = 3 \times (-2)^2 + 1 = 13$

Answer: $(-13, 13)$

7 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 6 m s^{-1} from the top of a cliff and hits the sea after 4 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$

$s = 6 \times 4 + \frac{1}{2} \times 9.8 \times 4^2$

$s = 24 + 78.4$

Answer: 102.4 m

8 Area between a line and a curve (All boards)

The curve $y = x^2$ and the line $y = 3x$ meet at the origin and at P.
Find the area enclosed between them.

$x^2 = 3x$ gives $x = 0$ or $x = 3$

Area $= \int_0^3 (3x - x^2)\,dx = [\frac{3x^2}{2} - \frac{x^3}{3}]$ from 0 to 3

$= \frac{3^3}{2} - \frac{3^3}{3} = \frac{9}{2}$

Answer: $\frac{9}{2}$

9 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 5 and 15 minutes.

Frequency = frequency density × class width
$5 \leq t < 15$: $2 \times 10 = 20$
Answer: 20

10 Maximum value from R form (All boards)

$f(x) = 3 \sin x + 4 \cos x + 6$.
Write f(x) in the form $R \sin(x + \alpha) + 6$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{3^2 + 4^2} = 5$, $\tan \alpha = \frac{4}{3}$ so $\alpha = 53.1°$

$f(x) = 5 \sin(x + 53.1°) + 6$
$\sin(x + \alpha)$ has maximum value 1, so f(max) = 5 + 6
Answer: f(x) = 5 sin(x + 53.1°) + 6; maximum 11