Day 13 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $11 \leq 3x + 2 < 17$.

Subtract 2 from each part: $9 \leq 3x < 15$
Divide each part by 3.
Answer: $3 \leq x < 5$

2 Natural logs and e^x (All boards)

Solve $e^{2x+2} = 12$.
Give your answer to 3 significant figures.

Take natural logs: $2x + 2 = \ln 12 = 2.485$
$x = 0.242$
Answer: $x = 0.242$

3 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of size 20 is taken from 100 students: 45 Under 30, 40 30 to 50, 15 Over 50.
How many 30 to 50 should be in the sample?

$$\text{Number} = 20 \times \frac{40}{100}$$
$$= 8$$
Answer: 8

4 Connected rates of change (All boards)

Water fills a tank with a square base of side 4 m.
The depth h m increases at 0.1 m/min. Find the rate of increase of volume.

$V = 4^2h = 16h$, so $\frac{dV}{dh} = 16$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = 16 \times 0.1$$
Answer: 1.6 m^3/min

5 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 7 terms of the geometric series $4 + 8 + 16 + ...$

$a = 4$, $r = 2$, $n = 7$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{4(2^7 - 1)}{2 - 1}$$
$$= 508$$
Answer: 508

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 3.25 m s^{-1}.
At that instant car Q starts from rest at O with constant acceleration 0.5 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $3.25\,t = \frac{1}{2} \times 0.5 \times t^2$

$t(3.25 - 0.25\,t) = 0$, so $t = 0$ or $t = 3.25 \div 0.25$
Answer: 13 s

7 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{3x - 9}{(x - 4)(x - 5)}$ in partial fractions.

Write $\frac{3x - 9}{(x - 4)(x - 5)} = \frac{A}{(x - 4)} + \frac{B}{(x - 5)}$

$3x - 9 = A(x - 5) + B(x - 4)$

$x = 4$: $3 = A(-1)$ so $A = -3$

$x = 5$: $6 = B(1)$ so $B = 6$

Answer: $\mathbf{\frac{-3}{(x - 4)} + \frac{6}{(x - 5)}}$

8 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 6 kg and length 6 m is supported by a pivot 0.25 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

$m\,g \times 0.25 = 6\,g \times (3 - 0.25)$

$m = \frac{6 \times 2.75}{0.25}$

Answer: 66 kg

9 Solve trig equation (calculator, degrees) (All boards)

Solve $\cos x = -0.40$ for $0° \leq x \leq 360°$.

Give your answers to 1 decimal place.

Reference angle (positive value): $\cos^{-1}(0.400) = 66.4°$

cos is negative in the second and third quadrants.

$x = 113.6°$ or $x = 246.4°$

Answer: x = 113.6°, 246.4°

10 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.4. If so, the probability that Sam is late is 0.6; otherwise it is 0.1.

Given that Sam is late, find the probability that it rains.

P(both) = 0.4 × 0.6 = 0.24; P(not first and second) = 0.6 × 0.1 = 0.06

P(second) = 0.24 + 0.06 = 0.3

P = 0.24 ÷ 0.3

Answer: $\mathbf{\frac{4}{5} = 0.8}$