Day 129 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Maximum value from R form (All boards)

f(x) = 20 sin x + 21 cos x + 1.
Write f(x) in the form R sin(x + α) + 1, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{20^2 + 21^2} = 29$, $\tan \alpha = \frac{21}{20}$ so $\alpha = 46.4°$

f(x) = 29 sin(x + 46.4°) + 1
sin(x + α) has maximum value 1, so f(max) = 29 + 1
Answer: f(x) = 29 sin(x + 46.4°) + 1; maximum 30

2 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(7, 1) and B(6, 6), in the form ax + by + c = 0, where a, b and c are integers.

Gradient $= \frac{6 - 1}{6 - 7} = -5$
$y - 1 = -5(x - 7)$
Multiply out and rearrange to integer coefficients.
Answer: 5x + y − 36 = 0

3 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.
Use the small angle approximations to find an approximation for 5 sin θ + cos θ.

$\sin \theta \approx \theta$ and $\cos \theta \approx 1 - \frac{\theta^2}{2}$

So $5 \sin \theta + \cos \theta \approx 1 + 5\theta - \frac{\theta^2}{2}$

Answer: $1 + 5\theta - \frac{\theta^2}{2}$

4 Friction: constant speed, find the coefficient (All boards)

A crate of mass 2 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 5.88 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the crate and ground.

R = 2 × 9.8 = 19.6
Constant speed: F = P = 5.88
F = μR: 5.88 = μ × 19.6
Answer: 0.3

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

30 pairs of data on daily mean temperature and sunshine hours have r = −0.26.
Test, at the 5% significance level, for evidence of negative correlation. The critical value is −0.3061.

$H_0: \rho = 0$, $H_1: \rho < 0$
r = −0.26 is not beyond the critical value −0.3061
Do not reject H_0
Answer: Do not reject H_0: there is not enough evidence of negative correlation

6 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(4 + 2x)^5$.

Term in x^2: ${}^5C_2 \times 4^3 \times (2x)^2$
= 10 × 64 × 4
Answer: 2560

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7 Normal approximation to the binomial (CIE)

X ~ B(50, 0.2). Use a normal approximation to find P(X ≤ 9), to 3 s.f.

np = 10 and npq = 8, so use Y ~ N(10, 8)
Continuity correction: P(Y < 9.5)
$z = (9.5 - 10) \div \sqrt{8}$
Answer: 0.430

8 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(4\mathbf{i} - 4\mathbf{j})$ N and $(5\mathbf{i} + 7\mathbf{j})$ N act on a particle of mass 3 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(4\mathbf{i} - 4\mathbf{j}) + (5\mathbf{i} + 7\mathbf{j})$
$= (9\mathbf{i} + 3\mathbf{j})$ N
F = ma: $a = \frac{1}{3}(9\mathbf{i} + 3\mathbf{j})$
Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

9 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 6x^2$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 12x$
dy/dx = 0 gives x = 0 or x = 4
Substitute: y = 0 at x = 0 and y = −32 at x = 4
Answer: A (0, 0), B (4, −32)

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -\frac{1}{2}$, $y = -x$

$= 1 + (-\frac{1}{2})(-x) + \frac{(-\frac{1}{2})((-\frac{3}{2}))}{2}(-x)^2$

$= 1 + \frac{1}{2}x + \frac{3}{8}x^2$

Valid when $|-x| < 1$, i.e. $|x| < 1$

Answer: $1 + \frac{1}{2}x + \frac{3}{8}x^2$; $|x| < 1$