Day 128 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(12, p)$. H_0: $p = 0.5$, H_1: $p < 0.5$.
In a sample, $x = 2$. It is given that $P(X \leq 2) = 0.0193$.
Test H_0 at the 5% significance level.

Compare 0.0193 with 0.05
$0.0193 < 0.05$, so reject H_0
Answer: Reject H_0: evidence that p is less than 0.5

2 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$3x^2 + 4x + k = 0$ has real roots.

$b^2 - 4ac \geq 0$
$16 - 12k \geq 0$
Answer: $k \leq \frac{4}{3}$

3 Addition formulae: exact values (All boards)

By writing 15° as 45° − 30°, find the exact value of tan 15°.

$$\tan(45° - 30°) = \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \times \frac{\sqrt{3}}{3}}$$

Simplify and rationalise the denominator: $2 - \sqrt{3}$
Answer: $2 - \sqrt{3}$

4 Venn diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities for events A and B.
Find P(A | B).

$P(A \mid B) = P(A \cap B) \div P(B)$
$= 0.2 \div (0.2 + 0.25) = 0.2 \div 0.45$
Answer: $\frac{4}{9} = 0.444$

5 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 6x + 8$, $y = -2x + 5$.

$x^2 - 6x + 8 = -2x + 5$
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
Use $y = -2x + 5$ to find each y.
Answer: x = 1, y = 3 and x = 3, y = −1

6 Maximum value from R form (All boards)

$f(x) = 12 \sin x + 5 \cos x + 5$.
Write f(x) in the form $R \sin(x + \alpha) + 5$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{12^2 + 5^2} = 13$, $\tan \alpha = \frac{5}{12}$ so $\alpha = 22.6°$
$f(x) = 13 \sin(x + 22.6°) + 5$
$\sin(x + \alpha)$ has maximum value 1, so f(max) = 13 + 5
Answer: f(x) = 13 sin(x + 22.6°) + 5; maximum 18

7 Constant acceleration: finding the initial speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration 0.5 m s^{-2} travels 32 m in 8 s.
Find its initial speed.

$s = ut + \frac{1}{2}at^2$

$32 = 8u + \frac{1}{2} \times 0.5 \times 8^2$

$8u = 16$

Answer: 2 m s^{-1}

8 Quadratic in sin or cos (All boards)

Solve $2\cos^2 x - \cos x = 0$ for $0° \leq x \leq 360°$.

Factorise: $(2\cos x - 1)\cos x = 0$ (as a quadratic in cos x)

$\cos x = \frac{1}{2}$ or $\cos x = 0$

$x = 60°, 90°, 270°, 300°$

Answer: x = 60°, 90°, 270°, 300°

9 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 3.

Centre (5, 3): $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 5)^2 + (y - 3)^2 = 9$

10 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A brick is dropped from rest from the top of a cliff of height 4.9 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $4.9 = \frac{1}{2} \times 9.8 \times t^2$

$4.9 = 4.9t^2$, so $t^2 = 1$

(b) $v = u + at = 9.8 \times 1$

Answer: (a) 1 s (b) 9.8 m s^{-1}