Day 127 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(4 + x)^8$.

Term in x^3: ${}^8C_3 \times 4^5 \times (1\ x)^3$
$= 56 \times 1024 \times 1$
Answer: 57344

2 Integrate standard functions (All boards)

Find
(a) $\int 12 \sin 4x\ dx$
(b) $\int 12 \cos 4x\ dx$

Use the standard integrals and divide by the coefficient of x where needed.
(a) $= -3 \cos 4x + c$
(b) $= 3 \sin 4x + c$
Answer: (a) $-3 \cos 4x + c$ (b) $3 \sin 4x + c$

3 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of the 16 members of one family who have the flu, which spreads easily.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here the people are not independent.
Answer: No, the people are not independent

4 Lift: tension in the cable (All boards)

A lift of mass 1000 kg is lowered by a vertical cable with constant acceleration 0.75 m s^{-2} downwards. Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolving downwards: $1000 \times 9.8 - T = 1000 \times 0.75$
$T = 9800 - 750$
Answer: 9050 N

5 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 1)^2 + (y - 3)^2 = 16$.
Determine whether the point P(−6, −7) is inside, outside or on the circle.

Distance^2 from the centre (1, 3) to P:
$(-6 - 1)^2 + (-7 - 3)^2 = 49 + 100 = 149$
$r^2 = 16$, and $149 > 16$.
Answer: P is outside the circle

6 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.1, H_1: $p \neq 0.1$. In a sample, x = 5.
Test at the 10% significance level.

Two-tailed test, so compare with $0.1 \div 2 = 0.05$
$P(X \geq 5) = 0.0127$
$0.0127 < 0.05$, so reject H_0
Answer: Reject H_0 ($P(X \geq 5) = 0.0127$)

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7 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(6\mathbf{i} + 29.4\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 29.4\,t - 4.9t^2$

$t = \frac{29.4}{4.9} = 6$

(b) $0 = 29.4^2 - 2 \times 9.8 \times H$

$H = \frac{864.36}{19.6}$

Answer: (a) 6 s (b) 44.1 m

8 Identity then solve (sin² to cos) (All boards)

Solve $-\sin^2 x - \cos x + 1 = 0$
for $0° \le x \le 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $\cos^2 x - \cos x = 0$
Factorise and solve: cos x = 0 or cos x = 1
x = 0°, 90°, 270°, 360°
Answer: x = 0°, 90°, 270°, 360°

9 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are x = 1 and y = −2, so a = 1 and c = −2.

Through (0, −5): $-5 = \frac{k}{0 - 1} + -2$

$\frac{k}{-1} = -3$, so k = 3

Answer: a = 1, c = −2, k = 3

10 Maximum value from R form (All boards)

f(x) = 20 sin x + 21 cos x + 6.
Write f(x) in the form R sin(x + α) + 6, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{20^2 + 21^2} = 29$, $\tan\alpha = \frac{21}{20}$ so α = 46.4°

f(x) = 29 sin(x + 46.4°) + 6
sin(x + α) has maximum value 1, so f(max) = 29 + 6
Answer: f(x) = 29 sin(x + 46.4°) + 6; maximum 35