Day 126 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Constant acceleration: s = ut + ½at² (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration 4 m s^{-2}. Its initial speed is 1 m s^{-1}. Find the distance travelled in 9 s.

$s = ut + \frac{1}{2}at^2$

$s = 1 \times 9 + \frac{1}{2} \times 4 \times 9^2$

Answer: 171 m

2 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(−5, −4) and B(2, 1), in the form ax + by + c = 0, where a, b and c are integers.

Gradient $= \frac{1-(-4)}{2-(-5)} = \frac{5}{7}$

$y - (-4) = \frac{5}{7}(x - (-5))$

Multiply out and rearrange to integer coefficients.

Answer: 5x − 7y − 3 = 0

3 Stationary points of a cubic (All boards)

$y = x^3 + 3x^2 - 9x + 1$.

Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 + 6x - 9 = 3(x + 3)(x - 1) = 0$

$x = -3$: $y = 28$; $x = 1$: $y = -4$

$d^2y/dx^2 = 6x + 6$: at $x = -3$ it is -12 (maximum), at $x = 1$ it is 12 (minimum)

Answer: (−3, 28) maximum; (1, −4) minimum

4 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each time a spinner is spun the probability of red is 0.4.

Find the probability of getting red at least once in 4 spins, to 3 s.f.

$P(\text{zero}) = 0.6^4$

$P(\text{at least one}) = 1 - 0.6^4$

Answer: 0.870

5 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{20} (6r + 8)$.

This is an arithmetic series with first term 14 and common difference 6.

$S_{20} = \frac{20}{2}[2 \times 14 + 19 \times 6]$

$= 1420$

Answer: 1420

6 Rewrite a sin x + b cos x in R form (All boards)

Express 24 sin x + 7 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°.

Give α to 1 decimal place.

$R \sin(x + \alpha) = R \sin x \cos \alpha + R \cos x \sin \alpha$

$R \cos \alpha = 24$ and $R \sin \alpha = 7$

$R = \sqrt{24^2 + 7^2} = 25$, $\tan \alpha = \frac{7}{24}$, $\alpha = 16.3°$

Answer: 25 sin(x + 16.3°)

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7 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.6. If so, the probability that an item is faulty is 0.7; otherwise it is 0.4. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.6 × 0.7 = 0.42; P(not first and second) = 0.4 × 0.4 = 0.16

P(second) = 0.42 + 0.16 = 0.58

P = 0.42 ÷ 0.58

Answer: $\frac{21}{29}$ = **0.724**

8 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 1 : 3.

Find AB and OP in terms of $\mathbf{a}$ and $\mathbf{b}$.

AB = OB − OA = $\mathbf{b} - \mathbf{a}$

AP = $\frac{1}{4}$ AB = $\frac{1}{4}(\mathbf{b} - \mathbf{a})$

OP = OA + AP = $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a})$

= $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: AB = $\mathbf{b} - \mathbf{a}$; OP = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

9 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + 6x^2 - 2x - 6$

(a) Show that $(x - 1)$ is a factor of f(x).

(b) Factorise f(x) completely.

f(1) = 2 + 6 + (−2) + (−6) = 0, so $(x - 1)$ is a factor.

$f(x) = (x - 1)(2x^2 + 8x + 6)$

$2x^2 + 8x + 6 = 2(x + 3)(x + 1)$

Answer: (b) $f(x) = 2(x + 3)(x + 1)(x - 1)$

10 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a train over 21 seconds as it moves in a straight line.

(a) Find its acceleration in the first 4 s.

(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{20}{4}$

(b) distance = area of trapezium

= $\frac{1}{2}(12 + 21) \times 20$

Answer: (a) 5 m s^{-2} (b) 330 m