Day 123 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 23$ and $Q_3 = 39$.

A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.

Determine whether 0 and 2 are outliers.

IQR = 39 − 23 = 16, so 1.5 × IQR = 24

Lower limit = 23 − 24 = −1; upper limit = 39 + 24 = 63

0 is not an outlier; 2 is not an outlier

Answer: 0 is not an outlier; 2 is not an outlier

2 Tension in a string supporting a hanging mass (All boards)

A particle of mass 8 kg hangs from a light string and is raised vertically with acceleration 0.5 m s^{-2}.

Taking g = 9.8 m s^{-2}, find the tension.

$T - mg = ma$

$T = 8 \times 9.8 + 8 \times 0.5$

Answer: 82.4 N

3 Arc length and perimeter of a sector (All boards)

A sector has radius 10 cm and angle $\frac{\pi}{2}$ radians.

Find the arc length, leaving π in your answer, and the perimeter.

Arc length = $r\theta = 10 \times \frac{\pi}{2} = 5\pi$ cm

Perimeter = arc + 2r = $5\pi + 20$ cm

Answer: Arc 5π cm; perimeter $(5\pi + 20)$ cm

4 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show, by a counter-example, that the statement "If n^2 is a multiple of 50 then n is a multiple of 50" is false, where n is a positive integer.

Look for n with n^2 divisible by 50 but n not divisible by 50.

Try n = 10: $n^2 = 100 = 50 \times 2$, a multiple of 50.

But 10 is not a multiple of 50, so the statement is false.

Answer: n = 10

5 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 8 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{4}{5}$. Take g = 9.8 m s^{-2}.

Find (a) its acceleration, (b) its speed after 3 s.

(a) Parallel to slope: $mg\sin\alpha = ma$

$a = 9.8 \times \frac{4}{5}$

(b) $v = u + at = 0 + 7.84 \times 3$

Answer: (a) 7.84 m s^{-2} (b) 23.52 m s^{-1}

6 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 6 terms of the geometric series 1 + 4 + 16 + ...

a = 1, r = 4, n = 6

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(4^6 - 1)}{4 - 1}$

= 1365

Answer: 1365

7 Identity then solve (sin² to cos) (All boards)

Solve $-6\sin^2 x - 3\cos x + 3 = 0$
for $0° \leq x \leq 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $6\cos^2 x - 3\cos x - 3 = 0$

Factorise and solve: $\cos x = -\frac{1}{2}$ or $\cos x = 1$

$x = 0°, 120°, 240°, 360°$

Answer: x = 0°, 120°, 240°, 360°

8 Grouped data: estimated mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.
Class: $20 \leq x < 25$; $25 \leq x < 30$; $30 \leq x < 35$; $35 \leq x < 40$
Frequency: 7, 7, 8, 12
Estimate the mean, to 3 s.f.

Midpoints: 22.5, 27.5, 32.5, 37.5
$\Sigma fx = 7\times22.5 + 7\times27.5 + 8\times32.5 + 12\times37.5 = 1060$
Mean $\approx 1060 \div 34$
Answer: 31.2

9 Rewrite a sin x + b cos x in R form (All boards)

Express $15\sin x + 8\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.
Give α to 1 decimal place.

$R\sin(x + \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$
$R\cos\alpha = 15$ and $R\sin\alpha = 8$
$R = \sqrt{15^2 + 8^2} = 17$, $\tan\alpha = \frac{8}{15}$, $\alpha = 28.1°$

Answer: 17 sin(x + 28.1°)

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 3x)^{-2}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -2$, $y = -3x$

$= 1 + (-2)(-3x) + \frac{(-2)((-3))}{2}(-3x)^2$

$= 1 + 6x + 27x^2$

Valid when $|-3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 + 6x + 27x^2$; $|x| < \frac{1}{3}$