Day 120 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show, by a counter-example, that the statement "If n^2 is a multiple of 25 then n is a multiple of 25" is false, where n is a positive integer.

Look for n with n^2 divisible by 25 but n not divisible by 25.

Try n = 5: $n^2 = 25 = 25 \times 1$, a multiple of 25.

But 5 is not a multiple of 25, so the statement is false.

Answer: n = 5

2 Friction: acceleration along rough ground (All boards)

A block of mass 5 kg is pulled along rough horizontal ground by a horizontal force of 12.3 N.

The coefficient of friction is 0.2. Take g = 9.8 m s^{-2}. Find the acceleration.

$R = 5 \times 9.8 = 49$

$F = \mu R = 0.2 \times 49 = 9.8$

$P - F = ma$: $12.3 - 9.8 = 5a$

Answer: 0.5 m s^{-2}

3 Reciprocal trig functions: exact values (All boards)

Write down the exact values of

(a) cosec 30° (b) cosec 60°

(a) $\text{cosec } 30° = \frac{1}{\sin 30°} = \frac{1}{\frac{1}{2}} = 2$

(b) $\text{cosec } 60° = \frac{1}{\sin 60°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

Answer: (a) 2 (b) $\frac{2\sqrt{3}}{3}$

4 Maximum friction on a horizontal surface (All boards)

A box of mass 5 kg rests on rough horizontal ground. The coefficient of friction is 0.25.

Taking g = 9.8 m s^{-2}, find the maximum friction force that can act on the box.

$R = mg = 5 \times 9.8 = 49$

$F_{max} = \mu R = 0.25 \times 49$

Answer: 12.25 N

5 Binomial: cumulative probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(10, 0.4). Find P(X > 2), giving your answer to 3 s.f.

$P(X > 2) = 1 - P(X \leq 2)$

$P(X \leq 2) = 0.1673$

Answer: 0.833

6 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 5$.

Use the Newton-Raphson method with $x_0 = 1$ to find x_1,

giving your answer to 3 decimal places.

$f'(x) = e^x$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{-2.282}{2.718} = 1.8394$

Answer: $x_1 = 1.839$

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7 Grouped data: estimated mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.
Class: $50 \le x < 55$; $55 \le x < 60$; $60 \le x < 65$; $65 \le x < 70$
Frequency: 9, 10, 2, 6
Estimate the mean, to 3 s.f.

Midpoints: 52.5, 57.5, 62.5, 67.5
$\Sigma fx = 9\times52.5 + 10\times57.5 + 2\times62.5 + 6\times67.5 = 1577.5$
Mean $\approx 1577.5 \div 27$
Answer: 58.4

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve
$y = x^2 - 8x + 5$.
Find the value of k.

$x^2 - 8x + 5 = -4x + k$
$x^2 - 4x + (5 - k) = 0$
A tangent touches once: discriminant = 0
$(-4)^2 - 4(5 - k) = 0$
Answer: k = 1

9 Differentiate tan, sec, cosec, cot (All boards)

Differentiate $y = \tan 4x$ with respect to x.

Use the standard result: $d/dx(\tan kx) = k \sec^2 kx$
Answer: $dy/dx = 4 \sec^2 4x$

10 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(n + 5)^2 - (n - 5)^2$ is a multiple of 4 for every integer n.

$(n + 5)^2 - (n - 5)^2 = (n^2 + 10n + 25) - (n^2 - 10n + 25)$
$= 20n = 4 \times 5n$
5n is an integer, so the expression is a multiple of 4.
Answer: Shown