Day 12 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 2D vectors: magnitude and direction (All boards)

The vector p = 5**i** + 12**j**.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|p| = \sqrt{5^2 + 12^2} = 13$

(b) Acute angle = $\tan^{-1}(\frac{12}{5}) = 67.4°$

p is in the first quadrant, so the angle is 67.4°
Answer: (a) 13 (b) 67.4°

2 Sampling methods (AQA, Edexcel, OCR, MEI, Eduqas)

The 40 residents are numbered. A computer generates 15 different random numbers and those people are asked to take part.
Name the sampling method used.

This is simple random sampling: every person has the same chance of selection and the choice is made purely by random numbers.
Answer: simple random sampling

3 Moments: balancing a plank (AQA, Edexcel, OCR, MEI)

A light plank rests on a pivot. A child of mass 40 kg sits 1 m from the pivot on one side. Where must a child of mass 25 kg sit on the other side for the plank to balance horizontally?

Moments about the pivot:
40 g × 1 = 25 g × d
$d = \frac{40 \times 1}{25}$
Answer: 1.6 m from the pivot

4 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(115, 6^2)$. Given that P(X > a) = 0.75, find a to 3 s.f.

P(Z > z) = 0.75, so z = −0.6745 (from the inverse normal function)
a = 115 + −0.6745 × 6
Answer: 111

5 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos\alpha = \frac{4}{5}$. Find P and Q.

Horizontally: $P = 30\cos\alpha = 30 \times \frac{4}{5}$

Vertically: $Q = 30\sin\alpha = 30 \times \frac{3}{5}$

Answer: P = 24, Q = 18

6 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2 \times 5^x - 10$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $5^x \to 0$, so $y \to -10$: asymptote $y = -10$.
x = 0: $y = 2 \times 5^0 - 10 = -8$
y = 0: $2 \times 5^x = 10$, so $5^x = 5$ and x = 1
Answer: y = −10 ; (0, −8), (1, 0)

7 Connected rates of change (All boards)

The volume of a sphere increases at 16 cm^3/s.
Find the rate of increase of the radius when r = 4. Give your answer to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} = \frac{16}{4\pi \times 4^2} = 0.0796$

Answer: 0.0796 cm/s

8 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of three consecutive integers is a multiple of 3.

Let the integers be n, n + 1, ..., n + 2.
Sum = n + (n + 1) + (n + 2) = 3n + 3
= 3(n + 1), which is a multiple of 3.
Answer: Shown

9 Optimisation (algebraic set-up) (All boards)

A rectangle has perimeter 20 cm.
Find the maximum possible area of the rectangle and the dimensions that give it.

Sides x and 10 − x, so $A = x(10 - x) = 10x - x^2$
$dA/dx = 10 - 2x = 0$, so x = 5
$d^2A/dx^2 = -2 < 0$, so it is a maximum; A = 5 × 5
Answer: A = 25 cm^2, a 5 cm square

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 4x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + ...$ with $n = \frac{1}{2}$, $y = 4x$

$= 1 + \frac{1}{2}(4x) + \frac{\frac{1}{2}((-\frac{1}{2}))}{2}(4x)^2$

$= 1 + 2x - 2x^2$

Valid when $|4x| < 1$, i.e. $|x| < \frac{1}{4}$

Answer: $1 + 2x - 2x^2$; $|x| < \frac{1}{4}$