Day 118 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial: cumulative probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(20, 0.5). Find P(X ≤ 8), giving your answer to 3 s.f.

Using the cumulative binomial function (or adding P(X = 0) up to P(X = 8))

Answer: 0.252

2 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When $x^3 + kx^2 - 2x + 9$ is divided by (x + 1), the remainder is 8. Find the value of k.

$f(-1) = 8$

$-1 + k + 11 = 8$

$k = -2$

Answer: $k = -2$

3 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle starts from rest and moves in a straight line with constant acceleration 1.5 m s^{-2}.
It travels 3 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with u = 0

$3 = \frac{1}{2} \times 1.5 \times t^2$

$t^2 = 4$

Answer: 2 s

4 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(2 - \sqrt{5})^2$.

$= 2^2 - 2 \times 2 \times \sqrt{5} + (\sqrt{5})^2$

$= 4 - 4\sqrt{5} + 5$

Answer: $9 - 4\sqrt{5}$

5 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 5.25 m s^{-1}.

At that instant car Q starts from rest at O with constant acceleration 1.5 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $5.25\,t = \frac{1}{2} \times 1.5 \times t^2$

$t(5.25 - 0.75\,t) = 0$, so $t = 0$ or $t = 5.25 \div 0.75$

Answer: 7 s

6 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient $\frac{1}{2}$ and intercept 0.6.

Find n and a, giving a to 3 significant figures.

$\log y = \log a + n \log x$

Gradient $n = \frac{1}{2}$

$\log a = 0.6$ so $a = 10^{0.6} = 3.98$

Answer: $n = \frac{1}{2}$, $a = 3.98$

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7 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 3)^2 + (y - 2)^2 = 9$.
Determine whether the point P(5, −2) is inside, outside or on the circle.

Distance^2 from the centre (3, 2) to P:
$(5 - 3)^2 + (-2 - 2)^2 = 4 + 16 = 20$
$r^2 = 9$, and $20 > 9$.
Answer: P is outside the circle

8 Integrate using partial fractions (All boards)

$\frac{x + 6}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$.

Find A and B, and hence find $\int \frac{x + 6}{(x + 3)(x + 4)}\, dx$.

$x + 6 \equiv A(x + 4) + B(x + 3)$
$x = -3$ gives $A = 3$; $x = -4$ gives $B = -2$
$\int \left(\frac{3}{x + 3} + \frac{-2}{x + 4}\right) dx = 3 \ln|x + 3| + -2 \ln|x + 4| + c$
Answer: A = 3, B = −2; 3 ln |x + 3| − 2 ln |x + 4| + c

9 Normal approximation to the binomial (CIE)

X ~ B(80, 0.3). Use a normal approximation to find P(X ≤ 22), to 3 s.f.

np = 24 and npq = 16.8, so use Y ~ N(24, 16.8)
Continuity correction: P(Y < 22.5)
$z = (22.5 - 24) \div \sqrt{16.8}$
Answer: 0.357

10 Integration by parts (All boards)

Use integration by parts to find
$\int x\, e^{2x}\, dx$.

$u = x$, $dv/dx = e^{2x}$; $du/dx = 1$, $v = e^{2x}/2$
$\int u\, (dv/dx)\, dx = uv - \int v\, (du/dx)\, dx$
Evaluate the remaining integral and simplify
Answer: $\frac{e^{2x}(2x - 1)}{4} + c$