Day 117 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Solve trig equation (calculator, degrees) (All boards)

Solve $\sin x = -0.65$ for $0° \leq x \leq 360°$.
Give your answers to 1 decimal place.

Reference angle (positive value): $\sin^{-1}(0.650) = 40.5°$
sin is negative in the third and fourth quadrants.
$x = 220.5°$ or $x = 319.5°$
Answer: $x = 220.5°, 319.5°$

2 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 4 red and 5 blue sweets. Two are taken at random without replacement.
Find the probability that they are different colours.

$P(RB) = \frac{4}{9} \times \frac{5}{8}$, $P(BR) = \frac{5}{9} \times \frac{4}{8}$

Add: $\frac{5}{18} + \frac{5}{18}$

Answer: $\frac{5}{9}$

3 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points X and Y, 300 m apart, and cycle directly towards each other at constant speeds of 11 m s^{-1} and 9 m s^{-1}. Find the time until they meet.

Closing speed = $11 + 9 = 20$ m s^{-1}

Time = $\frac{300}{20}$

Answer: 15 s

4 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + (y + 5)^2 = 8$.
Write down the centre and the radius of the circle.

Compare with $(x - a)^2 + (y - b)^2 = r^2$.
Centre $(0, -5)$; $r^2 = 8$ so $r = 2\sqrt{2}$
Answer: Centre $(0, -5)$, radius $2\sqrt{2}$

5 Geometric distribution (CIE)

Seeds are planted one at a time until one grows, with probability 0.5 each time. X is the number planted.
Find P(X = 6), to 3 s.f.

$P(X = 6) = (1 - p)^5 \times p = (\frac{1}{2})^5 \times \frac{1}{2}$

Answer: 0.0156

6 Rearrange and iterate (All boards)

(a) Show that $x^3 - 3x - 3 = 0$ can be rearranged to $x = (3x + 3)^{\frac{1}{3}}$.
(b) Use x_{n+1} = this formula with $x_0 = 2$ to find x_1, x_2 to 3 d.p.

(a) $x^3 = 3x + 3$, so $x = \sqrt[3]{(3x + 3)}$
(b) $x_1 = 2.0801$, $x_2 = 2.0984$
Answer: (a) shown (b) $x_1 = 2.080$, $x_2 = 2.098$

7 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2 \times 5^x - 50$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $5^x \to 0$, so $y \to -50$: asymptote $y = -50$.
$x = 0$: $y = 2 \times 5^0 - 50 = -48$
$y = 0$: $2 \times 5^x = 50$, so $5^x = 25$ and $x = 2$
Answer: $y = -50$; $(0, -48)$, $(2, 0)$

8 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\sin A = \frac{7}{25}$.

Find the exact values of sin 2A and cos 2A.

$\cos A = \sqrt{1 - \sin^2 A} = \frac{24}{25}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{7}{25} \times \frac{24}{25} = \frac{336}{625}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{576}{625} - \frac{49}{625} = \frac{527}{625}$

Answer: $\sin 2A = \frac{336}{625}$, $\cos 2A = \frac{527}{625}$

9 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 19 and common ratio $\frac{1}{5}$.

Find the sum to infinity.

$|r| < 1$, so the series converges.

$S_\infty = \frac{a}{1 - r} = \frac{19}{1 - \frac{1}{5}}$

$= \frac{95}{4}$

Answer: $\frac{95}{4}$

10 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 4 kg and length 5 m is supported by a pivot 0.5 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

$m g \times 0.5 = 4 g \times (2.5 - 0.5)$

$m = \frac{4 \times 2}{0.5}$

Answer: 16 kg