Day 116 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $e^{5x} + \sin 5x$

(b) $5 \sin 5x$

(a) $dy/dx = 5\,e^{5x} + 5\cos 5x$

(b) $dy/dx = 25\cos 5x$

Answer: (a) $5\,e^{5x} + 5\cos 5x$ (b) $25\cos 5x$

2 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = \frac{5}{2}x - 4$.

Find the equation of the line perpendicular to l that passes through $(-1, 6)$.

Gradient of l is $\frac{5}{2}$, so the perpendicular gradient is $-\frac{2}{5}$.

$y - 6 = -\frac{2}{5}(x - (-1))$

$y = -\frac{2}{5}x + \frac{28}{5}$

Answer: $y = -\frac{2}{5}x + \frac{28}{5}$

3 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = e^{3x - 9}$
at the point where $x = 3$.

Use the chain rule: $dy/dx = 3\,e^{3x - 9}$

At $x = 3$: $dy/dx = 3$

Answer: 3

4 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(90, 3^2)$. Given that $P(X > a) = 0.6$, find a to 3 s.f.

$P(Z > z) = 0.6$, so $z = -0.2533$ (from the inverse normal function)

$a = 90 + -0.2533 \times 3$

Answer: 89.2

5 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos\alpha = \frac{3}{5}$. Find P and Q.

Horizontally: $P = 15\cos\alpha = 15 \times \frac{3}{5}$

Vertically: $Q = 15\sin\alpha = 15 \times \frac{4}{5}$

Answer: P = 9, Q = 12

6 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-2x - 6}{(x + 1)(x + 5)}$ in partial fractions.

Write $\frac{-2x - 6}{(x + 1)(x + 5)} = \frac{A}{(x + 1)} + \frac{B}{(x + 5)}$

$-2x - 6 = A(x + 5) + B(x + 1)$

$x = -1$: $-4 = A(4)$ so $A = -1$

$x = -5$: $4 = B(-4)$ so $B = -1$

Answer: $\frac{-1}{(x + 1)} + \frac{-1}{(x + 5)}$

7 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is $(2t + 4)$ m s^{-2}.
When t = 0 its velocity is 5 m s^{-1}.
Find its velocity when t = 2.

$v = \int a\,dt = t^2 + 4t + c$
$t = 0, v = 5$ gives $c = 5$
When t = 2: $v = 4 + 8 + 5$
Answer: 17 m s^{-1}

8 Rewrite a sin x + b cos x in R form (All boards)

Express $4 \sin x + 3 \cos x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.
Give α to 1 decimal place.

$R\cos(x - \alpha) = R\cos x \cos\alpha + R \sin x \sin\alpha$
$R\cos\alpha = 3$ and $R\sin\alpha = 4$
$R = \sqrt{4^2 + 3^2} = 5$, $\tan\alpha = \frac{4}{3}$, $\alpha = 53.1°$
Answer: $5\cos(x - 53.1°)$

9 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -3, -1$ and 4.
$f(x) = k(x + 3)(x + 1)(x - 4)$
At $(0, -24)$: $k \times 3 \times 1 \times (-4) = -24$
$k = 2$
Answer: $f(x) = 2(x + 3)(x + 1)(x - 4)$

10 Grouped data: estimated median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.
Class: $10 \leq x < 20$; $20 \leq x < 30$; $30 \leq x < 40$; $40 \leq x < 50$
Frequency: 12, 13, 10, 6
Estimate the median, to 3 s.f.

n = 41, so the median is the 20.5 th value, in class $20 \leq x < 30$
Cumulative frequency before this class = 12
$\text{Median} \approx 20 + \frac{(20.5 - 12)}{13} \times 10$
Answer: 26.5