Day 114 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(−6, 3) and B(6, −1), in the form ax + by + c = 0, where a, b and c are integers.

Gradient = $\frac{-1-3}{6-(-6)} = -\frac{1}{3}$

$y - 3 = -\frac{1}{3}(x - (-6))$

Multiply out and rearrange to integer coefficients.

Answer: $x + 3y - 3 = 0$

2 Exact trigonometric values (All boards)

Write down the exact value of sin 225°.

sin 225° has the same size as sin 45°, and the sign comes from the quadrant (CAST).

sin 45° = $\frac{\sqrt{2}}{2}$, so sin 225° = $-\frac{\sqrt{2}}{2}$

Answer: $-\frac{\sqrt{2}}{2}$

3 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.1, H_1: p < 0.1.

In a sample, x = 0. It is given that P(X ≤ 0) = 0.2059.

Test H_0 at the 10% significance level.

Compare 0.2059 with 0.1

0.2059 > 0.1, so do not reject H_0

Answer: Do not reject H_0: insufficient evidence that p is less than 0.1

4 Resolving a force into components (All boards)

A cable pulls a box with a force of 12 N at an angle of 30° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 12 cos 30°

Vertical = 12 sin 30°

Answer: horizontal 10.4 N, vertical 6.00 N

5 Partial fractions: find the constants (All boards)

Express $\frac{x-7}{(x+1)(x+5)}$ in the form $\frac{A}{x+1} + \frac{B}{x+5}$.

Find the values of A and B.

$x - 7 \equiv A(x + 5) + B(x + 1)$

x = −1: −8 = A(4), so A = −2

x = −5: −12 = B(−4), so B = 3

Answer: A = −2, B = 3

6 Grouped data: estimated median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.

Class: 20 ≤ x < 30; 30 ≤ x < 40; 40 ≤ x < 50; 50 ≤ x < 60

Frequency: 8, 16, 16, 16

Estimate the median, to 3 s.f.

n = 56, so the median is the 28 th value, in class 40 ≤ x < 50

Cumulative frequency before this class = 24

Median ≈ $40 + \frac{(28-24)}{16} \times 10$

Answer: 42.5

7 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that 6x + 8y = 21.

Assume that integers x and y exist with 6x + 8y = 21.
Then 2(3x + 4y) = 21, so 21 would be a multiple of 2.
But 21 = 2 × 10 + 1, so it is not a multiple of 2.
This is a contradiction, so no such integers exist.
Answer: Proved

8 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$.

Write down a vector equation of the line through the origin, parallel to this line.

Parallel lines have the same direction vector
The line passes through O, so the position vector is the zero vector
Answer: $\mathbf{r} = \mathbf{s}\begin{pmatrix} \mathbf{-2} \\ \mathbf{3} \\ \mathbf{2} \end{pmatrix}$

9 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(3\mathbf{i} + 3\mathbf{j})$ N and $(2\mathbf{i} - 13\mathbf{j})$ N act on a particle of mass 5 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(3\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} - 13\mathbf{j})$
= $(5\mathbf{i} - 10\mathbf{j})$ N
F = ma: a = $\frac{1}{5}(5\mathbf{i} - 10\mathbf{j})$
Answer: $\mathbf{(i - 2j)\ m\ s^{-2}}$

10 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch y = |2x + 6|. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where 2x + 6 = 0: x = −3
x = 0: y = |6| = 6
Answer: Vertex (−3, 0) ; y-axis (0, 6)