Day 112 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arc length and perimeter of a sector (All boards)

A sector of a circle has radius 8 cm and angle 1.6 radians.
Find (a) the arc length, (b) the perimeter of the sector.

(a) Arc length = $r\theta = 8 \times 1.6 = 12.8$ cm
(b) Perimeter = arc + 2r = 12.8 + 16 = 28.8 cm
Answer: (a) 12.8 cm (b) 28.8 cm

2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 3)^2 + (y - 2)^2 = 18$.
Write down the centre and the radius of the circle.

Compare with $(x - a)^2 + (y - b)^2 = r^2$.
Centre (3, 2); $r^2 = 18$ so $r = 3\sqrt{2}$
Answer: Centre (3, 2), radius $3\sqrt{2}$

3 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 3 red and 3 blue sweets. Two are taken at random without replacement.
Find the probability that at least one of them is red.

P(no red) = P(BB) = $\frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$

P(at least one red) = $1 - \frac{1}{5}$

Answer: $\frac{4}{5}$

4 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 10 terms of the series
$-4 - 2 + 0 + 2 + \ldots$

$a = -4$, $d = 2$, $n = 10$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{10}{2}[2(-4) + 9 \times 2]$

$= 50$
Answer: 50

5 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(5\mathbf{i} + 2\mathbf{j})$ N and $(-13\mathbf{i} + 10\mathbf{j})$ N act on a particle of mass 4 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(5\mathbf{i} + 2\mathbf{j}) + (-13\mathbf{i} + 10\mathbf{j})$
$= (-8\mathbf{i} + 12\mathbf{j})$ N

F = ma: $a = \frac{1}{4}(-8\mathbf{i} + 12\mathbf{j})$

Answer: $(-2\mathbf{i} + 3\mathbf{j})$ m s^{-2}

6 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{3}{64}x^2$ for $0 \le x \le 4$, and 0 otherwise.

Find E(X).

$E(X) = \int$ from 0 to 4 of $x\,f(x)\,dx = \int$ from 0 to 4 of $\frac{3}{64}x^3\,dx$

$= \frac{3}{64} \times \frac{4^4}{4}$
Answer: 3

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7 Definite integrals (exact) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{3}} 6 \sin 3x \, dx$.

$[-2 \cos 3x]$ from 0 to $\frac{\pi}{3}$

$= -2 \cos \pi - (-2 \cos 0) = 2 + 2$

Answer: 4

8 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A boat moves with constant velocity $(-4\mathbf{i} + 3\mathbf{j})$ m s^{-1}.
At time t = 0 its position vector is $(4\mathbf{i} - 2\mathbf{j})$ m.
(a) Find its position vector when t = 3.
(b) Find the distance it travels in 3 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (4\mathbf{i} - 2\mathbf{j}) + 3(-4\mathbf{i} + 3\mathbf{j})$

(b) speed $= \sqrt{4^2 + 3^2} = 5$ m s^{-1}
distance $= 5 \times 3$

Answer: (a) $(-8\mathbf{i} + 7\mathbf{j})$ m (b) 15 m

9 Solve basic trig equation (exact, degrees) (All boards)

Solve $\sin x = \frac{1}{2}$ for $0° \leq x \leq 360°$.

Reference angle: $\sin 30° = \frac{1}{2}$

sin is positive in the first and second quadrants.
$x = 30°, 150°$

Answer: x = 30°, 150°

10 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−1, −2) and B is (1, 0). AB is a diameter of a circle.
Find the equation of the circle.

Centre = midpoint of AB = (0, −1)

$r^2 = (1 - 0)^2 + (0 - (-1))^2 = 2$

Equation: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $x^2 + (y + 1)^2 = 2$