Day 111 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 F = ma: find the acceleration (All boards)

A horizontal force of 12 N acts on a crate of mass 20 kg on a smooth horizontal surface.
Find the acceleration.

F = ma
12 = 20 a
Answer: 0.6 m s^{-2}

2 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

The product moment correlation coefficient between hours of sunshine and ice cream sales is $r = -0.77$.
Interpret this value.

r is close to -1, so there is strong negative linear correlation.
As hours of sunshine increases, ice cream sales tends to decrease.
Answer: strong negative linear correlation

3 Compound growth: find n (All boards)

A population of 400 grows by 8% each year.
After how many whole years will it first exceed 1200?

$400 \times 1.08^n > 1200$, so $1.08^n > 3$
$n > \frac{\log 3}{\log 1.08} = 14.27$
The least whole number of years is 15
Answer: 15 years

4 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x - 4)^2 + 6$, $x \geq 0$.
Find the range of f.

The minimum of $(x - 4)^2$ is 0 at $x = 4$, which is allowed since $4 \geq 0$.
So $f(x) \geq 6$.
Answer: $f(x) \geq 6$

5 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each customer buys a coffee with probability 0.2.
Find the probability that at least one of 9 customers buys a coffee, to 3 s.f.

P(zero) = 0.8^9
P(at least one) = $1 - 0.8^9$
Answer: 0.866

6 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 8$) is $v = (-4t^2 + 32t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 32 - 8t$

Maximum when $a = 0$: $t = \frac{32}{8} = 4$

$v = 32 \times 4 - 4 \times 4^2$
Answer: 64 m s^{-1} at t = 4

7 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that if $3n + 6$ is even, then n is even, where n is an integer.

Assume n is odd, so $n = 2k + 1$ for an integer k.
$3n + 6 = 6k + 3 + 6 = 2(3k + 4) + 1$
This is odd, which contradicts $3n + 6$ being even.
So the assumption is false: n must be even.
Answer: Proved

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8 Integrate using partial fractions (All boards)

$\frac{-x+2}{(x+4)(x+2)} \equiv \frac{A}{x+4} + \frac{B}{x+2}$.

Find A and B, and hence find $\int \frac{-x+2}{(x+4)(x+2)}\,dx$.

$-x + 2 \equiv A(x + 2) + B(x + 4)$

$x = -4$ gives $A = -3$; $x = -2$ gives $B = 2$

$\int \left(\frac{-3}{x+4} + \frac{2}{x+2}\right) dx = -3 \ln|x + 4| + 2 \ln|x + 2| + c$

Answer: $A = -3, B = 2$; $-3 \ln|x + 4| + 2 \ln|x + 2| + c$

9 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t + 1, y = 4t^2 - 3$.

Find a Cartesian equation of the curve.

From $x = t + 1$: $t = x - 1$

Substitute into y: $y = 4(x - 1)^2 - 3$

Answer: $y = 4(x - 1)^2 - 3$

10 Quadratic in sin or cos (All boards)

Solve $2\cos^2 x - \cos x - 1 = 0$ for $0° \le x \le 360°$.

Factorise: $(2 \cos x + 1)(\cos x - 1) = 0$ (as a quadratic in cos x)

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

$x = 0°, 120°, 240°, 360°$

Answer: $x = 0°, 120°, 240°, 360°$