Day 109 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.1$, H_1: $p < 0.1$.
In a sample, $x = 0$. It is given that $P(X \leq 0) = 0.0718$.
Test H_0 at the 5% significance level.

Compare 0.0718 with 0.05
$0.0718 > 0.05$, so do not reject H_0
Answer: Do not reject H_0: insufficient evidence that p is less than 0.1

2 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(4 + \sqrt{5})(4 - \sqrt{5})$.

$= 4^2 - (\sqrt{5})^2$
$= 16 - 5$
Answer: 11

3 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length $= r\theta = 10 \times \frac{3\pi}{4} = \frac{15\pi}{2}$ cm

(b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times \frac{3\pi}{4} = \frac{75\pi}{2}$ cm^2

Answer: (a) $\frac{15\pi}{2}$ cm (b) $\frac{75\pi}{2}$ cm^2

4 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of $X \sim N(30, 10^2)$.
a and b are equal distances from the mean. The shaded area, $P(X > b)$, is 0.35.
Find $P(a < X < b)$.

By symmetry $P(X < a) = 0.35$
$P(a < X < b) = 1 - 2 \times 0.35$
Answer: 0.3

5 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations
$x = 2t + 2$, $y = 2t^2 + 4$.
The point P has y-coordinate 22 and positive t.
Find the coordinates of P.

$2t^2 + 4 = 22$, so $t^2 = 9$, giving $t = 3$ ($t > 0$)
$x = 2 \times 3 + 2 = 8$
Answer: (8, 22)

6 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 8$) is $v = (-t^2 + 8t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 8 - 2t$

Maximum when $a = 0$: $t = \frac{8}{2} = 4$

$v = 8 \times 4 - 1 \times 4^2$
Answer: 16 m s^{-1} at t = 4

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7 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \sin^2 x \, dx$.

$\cos 2\theta = 1 - 2\sin^2\theta$, so $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Integrate term by term

Answer: $\frac{x}{2} - \frac{\sin 2x}{4} + c$

8 Car towing a caravan (All boards)

A car of mass 1000 kg tows a caravan of mass 500 kg on a level road with acceleration 0.25 m s^{-2}. Resistances: 300 N on the car, 200 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 300 - 200 = 1500 \times 0.25$
Caravan: $T - 200 = 500 \times 0.25$
Answer: D = 875 N, T = 325 N

9 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$2x^2 + 4x + k = 0$ has real roots.

$b^2 - 4ac \geq 0$
$16 - 8k \geq 0$
Answer: $k \leq 2$

10 Differentiate tan, sec, cosec, cot (AQA, Edexcel, OCR, MEI, Eduqas)

Differentiate $y = \operatorname{cosec} 2x$ with respect to x.

Use the standard result: $d/dx(\operatorname{cosec} kx) = -k \operatorname{cosec} kx \cot kx$
Answer: $dy/dx = -2 \operatorname{cosec} 2x \cot 2x$