Day 108 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Evaluating fractional and negative indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $49^{-\frac{1}{2}}$.

$49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}}$

$49^{\frac{1}{2}} = 7$, so $49^{\frac{1}{2}} = 7^1 = 7$

$= \frac{1}{7}$

Answer: $\mathbf{\frac{1}{7}}$

2 Integrate (ax + b)^n and related (All boards)

Find $\int (5x + 1)^2 \, dx$.

Reverse the chain rule: raise the power by 1, divide by the new power and by 5

Here a = 5

Answer: $\mathbf{\frac{(5x + 1)^3}{15} + c}$

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(2x^4)^3 \div 2x$.

$(2x^4)^3 = 8x^{12}$

$8x^{12} \div 2x$: divide the numbers and subtract the powers

$= 4x^{11}$

Answer: $\mathbf{4x^{11}}$

4 Connected rates of change (All boards)

The volume of a sphere increases at 24 cm^3/s.

Find the rate of increase of the radius when r = 2. Give your answer to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} = \frac{24}{4\pi \times 2^2} = 0.477$

Answer: 0.477 cm/s

5 Particle held by a string and a horizontal force (All boards)

A ball of weight 9 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{4}{3}$. Find P and the tension T N.

Vertically: $T \cos\theta = 9$

Horizontally: $T \sin\theta = P$

Dividing: $P = 9 \tan\theta$; $T = \sqrt{9^2 + P^2}$

Answer: P = 12, T = 15

6 Mutually exclusive events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are mutually exclusive, with P(A) = 0.15 and P(B) = 0.45.

(a) Find $P(A \cup B)$.

(b) Find the probability that neither A nor B happens.

(a) Mutually exclusive, so $P(A \cap B) = 0$

$P(A \cup B) = 0.15 + 0.45 = 0.6$

(b) $1 - 0.6 = 0.4$

Answer: (a) 0.6 (b) 0.4

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7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 1$ and $y = 1$, so $a = 1$ and $c = 1$.

Through $(0, -2)$: $-2 = \frac{k}{0 - 1} + 1$

$\frac{k}{-1} = -3$, so $k = 3$

Answer: a = 1, c = 1, k = 3

8 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(15, p)$. H_0: $p = 0.3$, H_1: $p \neq 0.3$. In a sample, $x = 7$.

Test at the 5% significance level.

Two-tailed test, so compare with $0.05 \div 2 = 0.025$

$P(X \geq 7) = 0.131$

$0.131 > 0.025$, so do not reject H_0

Answer: Do not reject H_0 ($P(X \geq 7) = 0.131$)

9 Integrate using double angle formulae (All boards)

Use a double angle formula to find

$\int \sin x \cos x \, dx$.

$\sin 2\theta = 2 \sin \theta \cos \theta$, so $\sin x \cos x = \frac{1}{2} \sin 2x$

Integrate term by term

Answer: $-\frac{\cos 2x}{4} + c$

10 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a train over 43 seconds as it moves in a straight line.

(a) Find its acceleration in the first 5 s.

(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{16}{5}$

(b) distance = area of trapezium

$= \frac{1}{2}(30 + 43) \times 16$

Answer: (a) 3.2 m s^{-2} (b) 584 m