Day 105 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Degrees and radians (All boards)

(a) Convert $\frac{\pi}{2}$ radians to degrees.

(b) Convert 75° to radians, giving your answer in terms of π.

(a) $\frac{\pi}{2} \times \frac{180}{\pi} = 90°$

(b) $75 \times \frac{\pi}{180} = \frac{5\pi}{12}$

Answer: (a) 90° (b) $\frac{5\pi}{12}$ radians

2 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 5)(x - 3)(x - 5)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -5, 3, 5$

$x = 0$: $y = 5 \times (-3) \times (-5) = 75$

Answer: (−5, 0), (3, 0), (5, 0), (0, 75)

3 Car towing a caravan (All boards)

A car of mass 800 kg tows a caravan of mass 1000 kg on a level road with acceleration 1 m s^{-2}. Resistances: 200 N on the car, 200 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 200 - 200 = 1800 \times 1$

Caravan: $T - 200 = 1000 \times 1$

Answer: D = 2200 N, T = 1200 N

4 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$, with $u_1 = 1$ and $u_2 = 2$. Find u_6.

$u_3 = 2 + 1 = 3$

$u_4 = 3 + 2 = 5$, $u_5 = 5 + 3 = 8$

$u_6 = 8 + 5 = 13$

Answer: $u_6 = 13$

5 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 values, $\Sigma x = 1000$ and $\Sigma x^2 = 25800$.
Find the mean and the standard deviation.

Mean $= 1000 \div 40 = 25$

Variance $= \frac{25800}{40} - 25^2 = 645 - 625 = 20$

Standard deviation $= \sqrt{20}$

Answer: mean = 25, standard deviation = 4.47

6 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve $y = x^2 - 5x - 4$.
Find the value of k.

$x^2 - 5x - 4 = -3x + k$

$x^2 - 2x + (-4 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(-4 - k) = 0$

Answer: k = −5

Day 105 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Integration by parts (All boards)

Use integration by parts to find
$\int \ln x \, dx$.

$u = \ln x$, $dv/dx = 1$; $du/dx = \frac{1}{x}$, $v = x$

$\int u \, (dv/dx) \, dx = uv - \int v \, (du/dx) \, dx$
Evaluate the remaining integral and simplify
Answer: $x \ln x - x + c$

8 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a ball is $(-9\mathbf{i} + 12\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{9^2 + 12^2} = \sqrt{225}$

(b) $\tan \theta = \frac{12}{9}$

$\theta = \tan^{-1}(\frac{12}{9})$

Answer: (a) 15 m s^{-1} (b) 53.1°

9 Find the angle of a sector (All boards)

A sector has radius 4 cm and arc length 6.00 cm.
Find the angle of the sector in radians and the area of the sector.

$\theta = \frac{\text{arc length}}{r} = \frac{6.00}{4} = 1.5$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 4^2 \times 1.5 = 12.0$ cm^2

Answer: $\theta = 1.5$ radians; area 12.0 cm^2

10 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 7 values: 17, 3, 14, 18, 11, 14, 18
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 95$, $\Sigma x^2 = 1459$, $n = 7$
Mean $= 95 \div 7 = 13.57$

Variance $= \frac{1459}{7} - (13.57)^2 = 24.24$

Standard deviation $= \sqrt{24.24}$
Answer: mean = 13.6, standard deviation = 4.92