Day 102 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Compound growth: find n (All boards)

A population of 250 grows by 2% each year.
After how many whole years will it first exceed 750?

$250 \times 1.02^n > 750$, so $1.02^n > 3$

$n > \dfrac{\log 3}{\log 1.02} = 55.48$

The least whole number of years is 56

Answer: 56 years

2 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{19} (5r + 8)$.

This is an arithmetic series with first term 13 and common difference 5.

$S_{19} = \dfrac{19}{2}[2 \times 13 + 18 \times 5]$

$= 1102$

Answer: 1102

3 Solve trig equation (calculator, degrees) (All boards)

Solve $\sin x = 0.70$ for $0° \leq x \leq 360°$.
Give your answers to 1 decimal place.

Reference angle: $\sin^{-1}(0.700) = 44.4°$
sin is positive in the first and second quadrants.
$x = 44.4°$ or $x = 135.6°$

Answer: $x = 44.4°, 135.6°$

4 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(n, p)$ has mean 18 and variance 4.5.
Find n and p.

$np = 18$ and $npq = 4.5$
$q = 4.5 \div 18 = 0.25$, so $p = 0.75$
$n = 18 \div 0.75$

Answer: $n = 24$, $p = 0.75$

5 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$, with
$u_1 = 2$ and $u_2 = 4$. Find u_6.

$u_3 = 4 + 2 = 6$
$u_4 = 6 + 4 = 10$, $u_5 = 10 + 6 = 16$
$u_6 = 16 + 10 = 26$

Answer: $u_6 = 26$

6 Energy: falling object (potential and kinetic energy) (CIE)

A particle of mass 2 kg is released from rest and falls 78.4 m. Take $g = 9.8$ m s^{-2}. Use energy to find (a) the loss in potential energy (3 s.f.), (b) the speed of the particle after falling 78.4 m.

(a) Loss in PE $= mgh = 2 \times 9.8 \times 78.4 = 1536.64$

(b) $\dfrac{1}{2} \times 2 \times v^2 = 1536.64$

$v^2 = 1536.64$

Answer: (a) 1540 J (b) 39.2 m s^{-1}

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7 Identity then solve (sin² to cos) (All boards)

Solve $-4\sin^2 x - 2\cos x + 2 = 0$
for $0° \le x \le 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $4\cos^2 x - 2\cos x - 2 = 0$

Factorise and solve: $\cos x = 1$ or $\cos x = -\frac{1}{2}$

$x = 0°, 120°, 240°, 360°$

Answer: x = 0°, 120°, 240°, 360°

8 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A golf ball is projected from level ground with velocity $(15\mathbf{i} + 29.4\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 29.4\,t - 4.9t^2$

$t = \frac{29.4}{4.9} = 6$

(b) $0 = 29.4^2 - 2 \times 9.8 \times H$

$H = \frac{864.36}{19.6}$

Answer: (a) 6 s (b) 44.1 m

9 Coding (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The values x are coded using $y = (x - 20) \div 10$.
The coded values have mean $\bar{y} = 3.2$ and standard deviation 1.1.
Find the mean and standard deviation of x.

$x = 20 + 10y$
Mean: $20 + 10 \times 3.2 = 52$
Standard deviation: $10 \times 1.1 = 11$ (adding 20 has no effect)
Answer: mean = 52, standard deviation = 11

10 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |x^2 - 36|$. Write down the coordinates where the graph meets the axes and the coordinates of the local maximum point.

$y = 0$: $x^2 = 36$, so $x = \pm 6$
$x = 0$: $y = |-36| = 36$. The part of $x^2 - 36$ below the x-axis is reflected in it.
The local maximum is at x = 0.
Answer: (6, 0), (−6, 0), (0, 36) ; maximum (0, 36)