Day 100 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Tension in a string supporting a hanging mass (All boards)

A particle of mass 2 kg hangs from a light string and is at rest. Taking g = 9.8 m s^{-2}, find the tension in the string.

No acceleration, so T = mg

T = 2 × 9.8

Answer: 19.6 N

2 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with days of growth from 12 to 24, the regression line is H = 10 + 2d.
Estimate plant height (cm) when d = 17 and comment on the reliability.

H = 10 + 2 × 17 = 44

17 is inside the range 12 to 24, so this is interpolation.

Answer: H = 44; estimate is reliable, as 17 is within the data range (interpolation)

3 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-1} and constant acceleration $(-\mathbf{i} + \mathbf{j})$ m s^{-2}.
Find its velocity after 5 s.

v = u + at

$v = (-2\mathbf{i} - 3\mathbf{j}) + 5(-\mathbf{i} + \mathbf{j})$

$= (-2\mathbf{i} - 3\mathbf{j}) + (-5\mathbf{i} + 5\mathbf{j})$

Answer: $(-7\mathbf{i} + 2\mathbf{j})$ m s^{-1}

4 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x - 3)^2 + 3$, $x \geq 0$.
Find the range of f.

The minimum of $(x - 3)^2$ is 0 at x = 3, which is allowed since $3 \geq 0$.

So $f(x) \geq 3$.

Answer: $f(x) \geq 3$

5 Addition formulae: exact values (All boards)

By writing 105° as 60° + 45°, find the exact value of sin 105°.

sin(60° + 45°) = sin 60° cos 45° + cos 60° sin 45°

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}$

Simplify: $\frac{\sqrt{6} + \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} + \sqrt{2}}{4}$

6 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - 2x^2 + 3x$.
C is reflected in the y-axis. Find the equation of the image.

Reflection in the y-axis: replace x by −x.

$y = -x^3 - 2x^2 - 3x$

Answer: $y = -x^3 - 2x^2 - 3x$

7 Solve with a shifted angle (All boards)

Solve $\sin(x + 30°) = -\frac{\sqrt{3}}{2}$ for $0° \leq x \leq 360°$.

Let u = x + 30°, so $30° \leq u \leq 390°$

$\sin u = -\frac{\sqrt{3}}{2}$: u = 240°, 300°

x = u − 30°: x = 210°, 270°

Answer: x = 210°, 270°

Day 100 **A Level Maths: C Tier 10-a-day: Model Solutions**

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve
$y = x^2 + 2x + 3$.
Find the value of k.

$x^2 + 2x + 3 = -4x + k$
$x^2 + 6x + (3 - k) = 0$
A tangent touches once: discriminant = 0
$(6)^2 - 4(3 - k) = 0$
Answer: $k = -6$

9 Definite integrals (exact) (All boards)

Evaluate

$\int_0^3 (x^2 + 3)\,dx$.

$[\frac{x^3}{3} + 3x]$ from 0 to 3
$= \frac{27}{3} + 9 = 18$
Answer: 18

10 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.2. If so, the probability that Sam is late is 0.6; otherwise it is 0.5.
Given that Sam is late, find the probability that it rains.

P(both) = 0.2 × 0.6 = 0.12; P(not first and second) = 0.8 × 0.5 = 0.4
P(second) = 0.12 + 0.4 = 0.52
P = 0.12 ÷ 0.52
Answer: $\frac{3}{13} = 0.231$