Day 10 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 2 ln x
(b) e^{2x} + sin 2x

(a) dy/dx = $\frac{2}{x}$

(b) dy/dx = 2 e^{2x} + 2 cos 2x

Answer: (a) $\frac{2}{x}$ (b) 2 e^{2x} + 2 cos 2x

2 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$, with
$u_1 = 3$ and $u_2 = 4$. Find u_6.

$u_3 = 4 + 3 = 7$
$u_4 = 7 + 4 = 11$, $u_5 = 11 + 7 = 18$
$u_6 = 18 + 11 = 29$
Answer: $u_6 = 29$

3 Compound growth: find n (All boards)

£5000 invested at 8% compound interest per year.
After how many whole years will it first exceed £7500?

$5000 \times 1.08^n > 7500$, so $1.08^n > 1.5$

$n > \frac{\log 1.5}{\log 1.08} = 5.268$

The least whole number of years is 6
Answer: 6 years

4 Two particles over a smooth pulley (All boards)

Two particles of masses 2 kg and 3 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take g = 9.8 m s^{-2}. Find the acceleration and the tension in the string.

3 kg: $3 \times 9.8 - T = 3a$
2 kg: $T - 2 \times 9.8 = 2a$
Adding: $9.8 = 5a$, then $T = 2(9.8 + a)$
Answer: a = 1.96 m s^{-2}, T = 23.52 N

5 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with days of growth from 6 to 18, the regression line is H = 19 + 4d.
Estimate plant height (cm) when d = 24 and comment on the reliability.

$H = 19 + 4 \times 24 = 115$
24 is outside the range 6 to 18, so this is extrapolation.
Answer: H = 115; estimate is unreliable, as 24 is outside the data range (extrapolation)

6 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(\mathbf{i} + 7\mathbf{j})$ N and $(-7\mathbf{i} + 2\mathbf{j})$ N act on a particle of mass 3 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(\mathbf{i} + 7\mathbf{j}) + (-7\mathbf{i} + 2\mathbf{j})$
$= (-6\mathbf{i} + 9\mathbf{j})$ N

F = ma: $a = \frac{1}{3}(-6\mathbf{i} + 9\mathbf{j})$

Answer: $(-2\mathbf{i} + 3\mathbf{j})$ m s^{-2}

7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(20, p). H_0: p = 0.5, H_1: p > 0.5.

Find the critical region for a test at the 10% significance level.

$P(X \geq 14) = 0.0577 \leq 0.1$

$P(X \geq 13) = 0.132 > 0.1$

Answer: $X \geq 14$

8 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (5, −1) and B is (7, 7).

Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (6, 3)

Gradient of AB = $\frac{8}{2} = 4$, so perpendicular gradient = $-\frac{1}{4}$

$y - 3 = -\frac{1}{4}(x - 6)$

Multiply out and rearrange.

Answer: x + 4y = 18

9 Arc length and perimeter of a sector (All boards)

A sector of a circle has radius 9 cm and angle 1.8 radians.

Find (a) the arc length, (b) the perimeter of the sector.

(a) Arc length = $r\theta = 9 \times 1.8 = 16.2$ cm

(b) Perimeter = arc + 2r = 16.2 + 18 = 34.2 cm

Answer: (a) 16.2 cm (b) 34.2 cm

10 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 4)^2 + (y + 4)^2 = 25$.

Determine whether the point P(−8, 6) is inside, outside or on the circle.

Distance^2 from the centre (4, −4) to P:

$(-8 - 4)^2 + (6 - (-4))^2 = 144 + 100 = 244$

$r^2 = 25$, and 244 > 25.

Answer: P is outside the circle