Day 99 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A sports club has 45 junior members, 65 senior members and 20 veteran members.
A stratified sample of size 26 is taken. Find the number selected from each group.

Total = 45 + 65 + 20 = 130

Sampling fraction = 26/130 = $\frac{1}{5}$

junior: 45 ÷ 5 = 9
senior: 65 ÷ 5 = 13
veteran: 20 ÷ 5 = 4

Answer: junior 9, senior 13, veteran 4

2 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $8\cos^2 x - 2\sin x - 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $1 - 8\sin^2 x - 2\sin x = 0$, i.e. $8\sin^2 x + 2\sin x - 1 = 0$

$(4\sin x - 1)(2\sin x + 1) = 0$, so $\sin x = \frac{1}{4}$ or $-\frac{1}{2}$

$\sin x = \frac{1}{4}$: $x = 14.5°, 165.5°$

$\sin x = -\frac{1}{2}$: $x = 210.0°, 330.0°$

Answer: x = 14.5°, 165.5°, 210.0°, 330.0°

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (8, 7) and (−2, −5).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint = $(\frac{8 + (-2)}{2}, \frac{7 + (-5)}{2}) = (3, 1)$

(b) Gradient = $\frac{-12}{-10} = \frac{6}{5}$

$y - 7 = \frac{6}{5}(x - 8)$

Multiply out and collect terms: $6x - 5y - 13 = 0$

Answer: (a) (3, 1) (b) 6x − 5y − 13 = 0

4 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -2 \\ -4 \\ 1 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 8 \\ -14 \\ 6 \end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 3.
Find the position vector of P.

$\text{AB} = \begin{pmatrix} 10 \\ -10 \\ 5 \end{pmatrix}$

$\text{AP} = \frac{2}{5}\text{AB} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 2 \\ -8 \\ 3 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{-8} \\ \mathbf{3} \end{pmatrix}$

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5 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{32}{243})^{-\frac{3}{5}}$.

$\frac{32}{243} = (\frac{2}{3})^5$, so the expression $= (\frac{2}{3})^{-3}$

$= \frac{27}{8}$

Answer: $\frac{27}{8}$

6 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = ln x + x − 2. Use the Newton-Raphson method with
$x_0 = 1.5$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 1 + \frac{1}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1.5 - \frac{-0.09453}{1.667} = 1.5567$

$x_2 = 1.5567 - \frac{-0.0006974}{1.642} = 1.5571$

Answer: $x_1 = 1.5567$, $x_2 = 1.5571$

7 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 14.7 m s^{-1} from a point 19.6 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-19.6 = 14.7\,t - 4.9t^2$

$4.9t^2 - 14.7\,t - 19.6 = 0$

$t = (14.7 \pm \sqrt{600.25}) \div 9.8 = (14.7 \pm 24.5) \div 9.8$

Positive root only.

Answer: 4 s

8 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + 18$ meets the circle $(x + 1)^2 + (y - 1)^2 = 50$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -3x + 18$: $(x + 1)^2 + (-3x + 17)^2 = 50$

$x^2 - 10x + 24 = 0$

$(x - 6)(x - 4) = 0$, so $x = 4$ or $x = 6$

$y = -3x + 18$: A = (4, 6), B = (6, 0)

$AB^2 = 2^2 + (-6)^2 = 40$

Answer: A(4, 6), B(6, 0), AB = $2\sqrt{10}$

9 Binomial mean and variance (All boards)

X has a binomial distribution with mean 6.25 and variance 4.6875.
(a) Find n and p.
(b) Find P(X = 6) to 3 significant figures.

(a) $np = 6.25$ and $np(1 - p) = 4.6875$

$1 - p = 4.6875 \div 6.25 = 0.75$, so $p = 0.25$ and $n = 6.25 \div 0.25 = 25$

(b) $P(X = 6) = {}^{25}C_6 \times 0.25^6 \times 0.75^{19} = 0.183$

Answer: (a) n = 25, p = 0.25 (b) 0.183

10 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A suitcase of mass 9 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{3}{5}$.

The coefficient of friction is 0.5. Take $g = 9.8$ m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 88.2 - \frac{4}{5}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{3}{5}T = 0.5(88.2 - \frac{4}{5}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.5 \times 9 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

Answer: 44.1 N