Day 98 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Union, conditional probability and independence (All boards)

P(A) = 0.6, P(B) = 0.4, P(A ∩ B) = 0.09.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.6 + 0.4 - 0.09 = 0.91$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.09}{0.4} = 0.225$

(c) $P(A)P(B) = 0.6 \times 0.4 = 0.24$, which does not equal $P(A \cap B)$

Answer: (a) 0.91 (b) 0.225 (c) not independent

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moves in a straight line with constant acceleration. Its initial speed is 11 m s^{-1}, its final speed is 21.5 m s^{-1} and the motion lasts 7 s.
Find the distance travelled.

$s = \frac{1}{2}(u + v)t$
$s = \frac{1}{2} \times (11 + 21.5) \times 7$
Answer: 113.75 m

3 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$.

Use $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4}$

$= [\frac{x}{2} + \frac{\sin 2x}{4}]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4}$

Answer: $\frac{\pi}{4}$

4 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (3, 1) and passes through the point (5, 5).
Find an equation of the circle.

$r^2 = (5 - 3)^2 + (5 - 1)^2$
$= 4 + 16 = 20$
Answer: $(x - 3)^2 + (y - 1)^2 = 20$

5 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown horizontally at 27 m s^{-1} from the top of a cliff 19.6 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $19.6 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 4$
t = 2 s
Horizontal: $x = 27 \times 2$
Answer: 2 s, 54 m

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 6?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(2, −4) becomes (2, 4)
(b) The maximum of the middle part is 4. Compare k = 6 with 4:
k > 4 gives 2 intersections with y = k.
Answer: (a) (2, 4) (b) 2

7 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 4 cm. Angle AOB = 1.8 radians.
Find the area of the segment, to 3 s.f.

Sector area = $\frac{1}{2} \times 4^2 \times 1.8 = 14.4$

Triangle area = $\frac{1}{2} \times 4^2 \times \sin 1.8 = 7.791$

Segment = sector − triangle = 6.609 cm^2
Answer: 6.61 cm^2

8 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.25, find the least number of components needed for the probability of at least one success to exceed 0.999.

P(at least one) = $1 - 0.75^n > 0.999$
$0.75^n < 0.001$
$n > \frac{\log 0.001}{\log 0.75} = 24.01$
Answer: n = 25

9 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 5 tan 3x with respect to x,

and find the exact gradient when $x = \frac{\pi}{9}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = 15 \sec^2 3x$

At $x = \frac{\pi}{9}$: $\frac{dy}{dx} = 60$

Answer: $\frac{dy}{dx} = 15 \sec^2 3x$; gradient 60

10 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 5 and common ratio 3.
Find the smallest value of n for which the sum of the first n terms exceeds 10000.

$S_n = \frac{5(3^n - 1)}{2} > 10000$
$3^n > 4001$
$n > \frac{\lg 4001}{\lg 3} = 7.550$
So the smallest integer n is 8
Answer: n = 8