Day 97 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t$, $y = \frac{3}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

Multiply the two equations: $xy = 2t \times \frac{3}{t} = 6$

So $y = \frac{6}{x}$

Answer: $y = \frac{6}{x}$

2 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 6 and Var(X) = 9.
Find (a) E(5X + 3),
(b) Var(5X + 3), (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 5 \times 6 + 3 = 33$
(b) $Var(aX + b) = a^2Var(X) = 25 \times 9 = 225$
(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 9 + 36 = 45$
Answer: (a) 33 (b) 225 (c) 45

3 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 24 N and 32 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 24 N force, to 1 d.p.

Magnitude = $\sqrt{(24^2 + 32^2)} = \sqrt{1600}$
$\tan \alpha = 32 \div 24$
Answer: 40 N at 53.1°

4 Fixed-point iteration (All boards)

(a) Show that $x^3 - 5x - 4 = 0$ can be written as $x = \sqrt[3]{5x + 4}$.
(b) Use $x_{n+1} = \sqrt[3]{5x_n + 4}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^3 = 5x + 4$, so $x = \sqrt[3]{5x + 4}$
(b) $x_1 = 2.4101$
$x_2 = 2.5225$
$x_3 = 2.5516$
Answer: $x_1 = 2.4101$, $x_2 = 2.5225$, $x_3 = 2.5516$

5 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{50}(3r + 5)$.

This is an arithmetic series with first term 8 and common difference 3
Last term = 155
$S = \frac{50}{2}(8 + 155) = 4075$
Answer: 4075

6 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 65 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 10 to t = 60.
The floors are 345 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (65 + 50) × V = 345
115 V = 690 so V = 6
Deceleration = 6 ÷ 5
Answer: V = 6; deceleration 1.2 m s^{-2}

7 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x - 1$ meets the circle $x^2 + (y + 3)^2 = 100$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = x - 1$: $x^2 + (x + 2)^2 = 100$

$x^2 + 2x - 48 = 0$

$(x - 6)(x + 8) = 0$, so $x = -8$ or $x = 6$

$y = x - 1$: A = $(-8, -9)$, B = $(6, 5)$

$AB^2 = 14^2 + 14^2 = 392$

Answer: A$(-8, -9)$, B$(6, 5)$, AB = $14\sqrt{2}$

8 Optimisation: open box and tank (All boards)

An open tank has a square base of side x m and volume 256 m^3.

Show that its surface area is $S = x^2 + \frac{1024}{x}$, and find the minimum value of S.

Height $h = \frac{256}{x^2}$, so $S = x^2 + 4xh = x^2 + \frac{1024}{x}$

$\frac{dS}{dx} = 2x - \frac{1024}{x^2} = 0$, so $x^3 = 512$ and $x = 8$

$\frac{d^2S}{dx^2} = 2 + \frac{2048}{x^3} > 0$, so minimum

$S = 64 + 128 = 192$ m^2

Answer: S = 192 m^2 (when x = 8)

9 Binomial one-tailed test (All boards)

A seed supplier says that 0.1 of seeds germinate. In a sample of 20 seeds, 5 germinate.
Test, at the 1% level, whether the true proportion is higher than claimed.

H_0: $p = 0.1$, H_1: $p > 0.1$. Let $X \sim B(20, 0.1)$ under H_0

$P(X \geq 5) = 1 - P(X \leq 4) = 0.0432$

$0.0432 > 0.01$, so do not reject H_0

There is insufficient evidence that the proportion is higher than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is higher

10 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ab^x$. A graph of ln y against x is a straight line with gradient ln 3 that passes through (0, ln 2).
Find the values of a and b.

$\ln y = \ln a + x \ln b$

gradient = $\ln b = \ln 3$, so $b = 3$

intercept $\ln a = \ln 2$, so $a = 2$

Answer: a = 2, b = 3