Day 96 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-x^2 - 12x - 42$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the maximum point of $y = -x^2 - 12x - 42$.

Take out the factor -1: $-x^2 - 12x - 42 = -(x^2 + 12x) - 42$

$= -[(x + 6)^2 - 36] - 42$

$= -(x + 6)^2 - 6$

$(x + 6)^2 \geq 0$ with equality at $x = -6$, so the maximum is -6 at $x = -6$.

Answer: (a) $-(x + 6)^2 - 6$ (b) $(-6, -6)$

2 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 10): $y = 3.2 + 0.8x$.

x is the advertising spend, y the weekly sales.

(a) Interpret the gradient.

(b) Estimate y when $x = 4$ and comment.

(a) The gradient: for each extra £100 of advertising, y is predicted to increase by 0.8

(b) $y = 3.2 + (0.8) \times 4 = 6.4$

$x = 4$ lies within the range of the data (interpolation), so the estimate is reliable

Answer: (a) y increases by 0.8 per extra £100 of advertising (b) 6.4, reliable (interpolation)

3 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^1 xe^{-2x}\,dx$.

Let $u = x$ and $\frac{dv}{dx} = e^{-2x}$, so $\frac{du}{dx} = 1$ and $v = -\frac{e^{-2x}}{2}$

Integral $= [uv] - \int v\frac{du}{dx}\,dx$

$= \left[-\frac{xe^{-2x}}{2}\right]_0^1 - \int_0^1 -\frac{e^{-2x}}{2}\,dx$

$= \frac{e^2 - 3}{4e^2}$

Answer: $\frac{e^2 - 3}{4e^2}$

4 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 3rd term 21 and 10th term 84.

(a) Find the first term and the common difference.

(b) Find the sum of the first 25 terms.

$a + 2d = 21$ and $a + 9d = 84$

Subtract: $7d = 63$, so $d = 9$ and $a = 3$

$S_{25} = \frac{25}{2}(2 \times 3 + 24 \times 9) = 2775$

Answer: (a) $a = 3$, $d = 9$ (b) 2775

5 Poisson: changing the interval (CIE)

Meteors are seen at random at an average rate of 0.5 per hour.

Find the probability that in 90 minutes there are exactly 0 (3 s.f.).

Rate in 90 minutes: $\lambda = 0.5 \times \frac{90}{60} = 0.75$

$X \sim Po(0.75)$

$P(X = 0) = e^{-0.75} \times \frac{0.75^0}{0!} = 0.472$

Answer: 0.472

6 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 21 cos θ + 20 sin θ ≡ 29 cos(θ − 43.6°),
solve 21 cos θ + 20 sin θ = 4 for 0 ≤ θ ≤ 360°. (1 d.p.)

29 cos(θ − 43.6°) = 4, so cos(θ − 43.6°) = $\frac{4}{29}$

θ − 43.6° = ±82.1°
θ = 43.6° + 82.1° = 125.7°
θ = 43.6° − 82.1° = −38.5°, so θ = 321.5°
Answer: θ = 125.7°, 321.5°

7 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 8 kg is moving on a smooth horizontal surface at 0 m s^{-1} when a constant horizontal force of 6 N, in the direction of motion, acts for 6 s.
Find the speed of the trolley at the end of this time and the distance it moves.

F = ma: a = 6 ÷ 8 = 0.75
v = u + at = 0 + 0.75 × 6
s = ut + ½at^2
Answer: 4.5 m s^{-1}; 13.5 m

8 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = x^2 − 6x + 11 for x ≤ 6.
Find the range of f.

f(x) = $(x − 3)^2$ + 2, with minimum at x = 3
x = 3 is in the domain, so the minimum value is 2.
Answer: f(x) ≥ 2

9 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 6 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.
A particle of mass 1 kg is attached 1.5 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: T_2(5) = 6 g(2.5) + 1 g(1)
T_2 = 31.36
T_1 + T_2 = 7 g so T_1 = 68.6 − 31.36
Answer: Near A: 37.24 N; near B: 31.36 N

10 Locating roots: change of sign (All boards)

The function f(x) = 2 cos x − x.
Show that f(x) = 0 has a root between x = 1 and x = 1.5 (x in radians).

f(1) = 0.0806
f(1.5) = −1.36
f is continuous and changes sign, so there is a root in the interval
Answer: f(1) > 0 and f(1.5) < 0: sign change