Day 95 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\sqrt{72} - \sqrt{2} - \sqrt{8}$, giving your answer in the form $k\sqrt{2}$.

$\sqrt{72} = 6\sqrt{2}$, $\sqrt{2} = \sqrt{2}$, $\sqrt{8} = 2\sqrt{2}$

$6\sqrt{2} - \sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$

Answer: $3\sqrt{2}$

2 Union, conditional probability and independence (All boards)

P(A) = 0.5, P(B) = 0.5, P(A ∩ B) = 0.07.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) P(A ∪ B) = 0.5 + 0.5 − 0.07 = 0.93

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.07}{0.5} = 0.14$

(c) P(A)P(B) = 0.5 × 0.5 = 0.25, which does not equal P(A ∩ B)

Answer: (a) 0.93 (b) 0.14 (c) not independent

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (−6, 1) and (−3, −3).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint $= \left(\frac{-6 + (-3)}{2}, \frac{1 + (-3)}{2}\right) = \left(-\frac{9}{2}, -1\right)$

(b) Gradient $= \frac{-4}{3} = -\frac{4}{3}$

$y - 1 = -\frac{4}{3}(x + 6)$

Multiply out and collect terms: 4x + 3y + 21 = 0

Answer: (a) $\left(-\frac{9}{2}, -1\right)$ (b) 4x + 3y + 21 = 0

4 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 20 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 4 kg is attached 2 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 20\,g(1) + 4\,g(1.5)$

$T_2 = 127.4$

$T_1 + T_2 = 24\,g$ so $T_1 = 235.2 - 127.4$

Answer: Near A: 107.8 N; near B: 127.4 N

5 Trig equations in radians (All boards)

Solve $\cos(x + 0.5) = 0.9$ for $0 \le x \le 2\pi$,
giving answers to 3 significant figures.

Let $\theta = x + 0.5$, so $\cos\theta = 0.9$ for $0.5 \le \theta \le 2\pi + 0.5$

Calculator: $\theta = 0.4510$, then use the symmetry of the graph

$\theta = 5.832, 6.734$

$x = \theta - 0.5$

$x = 5.332, 6.234$

Answer: x = 5.33, 6.23

Day 95 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t + 1$, $y = -t^2 - 2t + 2$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve in the form $y = f(x)$.

From the first equation: $t = \frac{x - 1}{2}$

Substitute t into $y = -t^2 - 2t + 2$ and expand.

$y = -\frac{1}{4}x^2 - \frac{1}{2}x + \frac{11}{4}$

Answer: $y = -\frac{1}{4}x^2 - \frac{1}{2}x + \frac{11}{4}$

7 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ab^x$. A graph of ln y against x is a straight line with gradient ln 3 that passes through (0, ln 6).
Find the values of a and b.

ln y = ln a + x ln b
gradient = ln b = ln 3, so b = 3
intercept ln a = ln 6, so a = 6
Answer: a = 6, b = 3

8 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A suitcase of mass 5 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{12}{13}$.

The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 49 - \frac{5}{13}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{12}{13}T = 0.2(49 - \frac{5}{13}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.2 \times 5 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)
Answer: 9.8 N

9 Total probability and Bayes (All boards)

R: has the condition; L: test is positive.
P(R) = 0.2, P(L | R) = 0.2, P(L | R') = 0.3.
(a) Find P(L).
(b) Find P(R | L).

(a) $P(L) = 0.2 \times 0.2 + 0.8 \times 0.3 = 0.04 + 0.24 = 0.28$

(b) $P(R \mid L) = \frac{0.2 \times 0.2}{0.28} = \frac{0.04}{0.28} = 0.143$

Answer: (a) 0.28 (b) $\frac{1}{7}$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ -4 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -2 \\ 0 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $0 + 0\lambda = -2 + (-2)\mu$

j: $-4 + (-2)\lambda = 0 + 2\mu$

Solving i and j: $\lambda = -1$, $\mu = -1$

k check: $-4 + (-2)(-1) = -2$ and $-4 + (-2)(-1) = -2$, equal

Point: $\begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{0} \\ \mathbf{-2} \\ \mathbf{-2} \end{pmatrix}$