Day 94 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$4x - 4 \leq -7$ and $x^2 + 2x - 8 \leq 0$.

$4x - 4 \leq -7$ gives $x \leq -\frac{3}{4}$

$x^2 + 2x - 8 \leq 0$: $(x + 4)(x - 2) \leq 0$ gives $-4 \leq x \leq 2$

Both must hold: $-4 \leq x \leq -\frac{3}{4}$

Answer: $-4 \leq x \leq -\frac{3}{4}$

2 Basic trig equations (degrees) (All boards)

Solve $\cos x = -\frac{\sqrt{2}}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = 135°$
Second solution: $360° - 135° = 225°$
Answer: x = 135°, 225°

3 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 3n + 10$ is even for all integers n.

$n^2 + 3n = n(n + 3)$
As 3 is odd, n and n + 3 have opposite parity,
so one of them is even and the product n(n + 3) is even.
Even + 10 = even, so $n^2 + 3n + 10$ is even.
Answer: n(n + 3) is even, so the expression is even

4 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 9 + 2\sin(\frac{\pi t}{6})$, where t is in hours after midnight.

Find (a) the greatest and least values, (b) the values of t with $0 \leq t \leq 12$ when it equals 8 m.

(a) sin ranges from -1 to 1: greatest $9 + 2 = 11$, least $9 - 2 = 7$

(b) $9 + 2\sin(\frac{\pi t}{6}) = 8$, so $\sin(\frac{\pi t}{6}) = -\frac{1}{2}$

$\frac{\pi t}{6} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so $t = 7$ or 11

Answer: (a) 11 m, 7 m (b) t = 7 and 11

5 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 45 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 10 to t = 30.

The floors are 195 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (45 + 20) × V = 195
65 V = 390 so V = 6
Deceleration = 6 ÷ 15
Answer: V = 6; deceleration 0.4 m s^{-2}

6 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.
A plot of $\log_{10} y$ against x is a straight line with intercept 0.602 and gradient 0.477.

Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$

$\log_{10} a = 0.602$, so $a = 10^{0.602} = 4.0$

$\log_{10} b = 0.477$, so $b = 10^{0.477} = 3.0$

Answer: a = 4.0, b = 3.0

7 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 2x)^{-2}$ up to the x^2 term.

(b) Use x = 0.03 to estimate $\frac{1}{1.06^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -2$, $a = 2$:

$= 1 - 4x + 12x^2$

(b) $1 + 2 \times 0.03 = 1.06$, so substitute x = 0.03:

$1 - 4 \times 0.03 + 12 \times 0.03^2 = 0.89080$

Answer: (a) $1 - 4x + 12x^2 + \ldots$ (b) 0.89080

8 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 5x$. Use the Newton-Raphson method with $x_0 = 0.3$ to find x_1 and x_2, to 4 d.p.

$f'(x) = e^x - 5$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 0.3 - \frac{-0.1501}{-3.650} = 0.2589$

$x_2 = 0.2589 - \frac{0.001126}{-3.705} = 0.2592$

Answer: $x_1 = 0.2589$, $x_2 = 0.2592$

9 Binomial two-tailed test (All boards)

A factory says a proportion 0.25 of its parts are defective. In a sample of 30 parts, 15 are defective. Test, at the 5% significance level, whether the proportion is different from 0.25.

H_0: $p = 0.25$, H_1: $p \neq 0.25$. Under H_0, $X \sim B(30, 0.25)$

$P(X \geq 15) = 1 - P(X \leq 14) = 0.0027$

Two-tailed: compare with 0.025. $0.0027 < 0.025$, so reject H_0

There is sufficient evidence that the proportion is different from 0.25

Answer: reject H_0: sufficient evidence that the proportion is different from 0.25

10 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A model boat moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2 + 3t)\mathbf{i} + (2t^3 - 4t)\mathbf{j}$ metres. Find the velocity when t = 2, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t + 3)\mathbf{i} + (6t^2 - 4)\mathbf{j}$

$t = 2$: $\mathbf{v} = (15)\mathbf{i} + (20)\mathbf{j}$

Speed $= \sqrt{(15^2 + 20^2)}$

Answer: $\mathbf{v} = (15)\mathbf{i} + (20)\mathbf{j}$ m s^{-1}; speed 25 m s^{-1}