Day 93 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - 4x + 5$.
C is reflected in the y-axis and then reflected in the x-axis.
Find its new equation.

Reflection in the y-axis: replace x by $-x$.
When reflected in the y-axis: $y = x^2 + 4x + 5$
Reflection in the x-axis: replace y by $-y$.
When reflected in the x-axis: $y = -x^2 - 4x - 5$
Answer: $y = -x^2 - 4x - 5$

2 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A boat moves with constant velocity $(6\mathbf{i} - 8\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(6^2 + -8^2)} = 10$
$\tan\theta = 8 \div 6$
Answer: 10 m s^{-1}; 53.1°

3 Laws of logarithms: solving equations (All boards)

Solve $2\ln x = \ln(x + 30)$.

$\ln x^2 = \ln(x + 30)$, so $x^2 = x + 30$
$x^2 - x - 30 = 0$, so $(x - 6)(x + 5) = 0$
$x > 0$ for $\ln x$ to exist, so $x = 6$
Answer: $x = 6$

4 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $x^2 + 6x + 14 > 0$ for all real values of x.

Complete the square: $x^2 + 6x + 14 = (x + 3)^2 + 5$
$(x + 3)^2 \geq 0$ for all real x, so $(x + 3)^2 \geq 0$.
Adding 5: the expression is ≥ 5, so > 0 for all real x.
Answer: $x^2 + 6x + 14 > 0$ for all x

5 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 18): $y = 60 - 2.5x$.
x is the temperature, y the gas used per day.
(a) Interpret the gradient.
(b) Estimate y when x = 17 and comment.

(a) The gradient: for each extra °C rise in temperature, y is predicted to decrease by 2.5
(b) $y = 60 + (-2.5) \times 17 = 17.5$
x = 17 lies within the range of the data (interpolation), so the estimate is reliable
Answer: (a) y decreases by 2.5 per extra °C rise in temperature (b) 17.5, reliable (interpolation)

6 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has second term 54 and fifth term 2.
Find (a) the common ratio, (b) the first term, (c) the sum to infinity.

$ar = 54$ and $ar^4 = 2$, so $r^3 = \frac{2}{54} = \frac{1}{27}$

$r = \frac{1}{3}$, and $a = 54 \div \frac{1}{3} = 162$

$S_\infty = \frac{a}{1 - r} = \frac{162}{1 - \frac{1}{3}} = 243$

Answer: (a) $r = \frac{1}{3}$ (b) $a = 162$ (c) 243

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7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 6 ml. Set mean 50 ml; 50 bottles have mean 52 ml.
Test at the 5% level whether the true mean is greater than 50.

H_0: $\mu = 50$, H_1: $\mu > 50$. Under H_0, $\overline{X} \sim N(50, \frac{6^2}{50})$

$z = \frac{52 - 50}{6/\sqrt{50}} = 2.357$

$P(Z > 2.357) = 0.0092 < 0.05$

Reject H_0: there is sufficient evidence that the mean has increased

Answer: reject H_0: sufficient evidence of an increase in the mean

8 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = x \ln x$,
and determine its nature.

$\frac{dy}{dx} = \ln x + 1 = 0$

$x = \frac{1}{e}$

$\frac{d^2y}{dx^2} = \frac{1}{x}$

At $x = \frac{1}{e}$: $\frac{d^2y}{dx^2} = e > 0$, so a minimum

Answer: $\left(\frac{1}{e}, -\frac{1}{e}\right)$ minimum

9 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards at 9.8 m s^{-1} from a point 14.7 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-14.7 = 9.8\,t - 4.9t^2$

$4.9t^2 - 9.8\,t - 14.7 = 0$

$t = (9.8 \pm \sqrt{384.16}) \div 9.8 = (9.8 \pm 19.6) \div 9.8$

Positive root only.

Answer: 3 s

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $4x^2y + 2xy + 2y^3 = 20$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(-1, 2)$.

Differentiate with respect to x: $4x^2\frac{dy}{dx} + 8xy + 2x\frac{dy}{dx} + 6y^2\frac{dy}{dx} + 2y = 0$

$\frac{dy}{dx} = \frac{y(-4x - 1)}{2x^2 + x + 3y^2}$

At $(-1, 2)$: gradient $= \frac{6}{13}$

Answer: $\frac{dy}{dx} = \frac{y(-4x - 1)}{2x^2 + x + 3y^2}$; gradient $\frac{6}{13}$