Day 92 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 8x + 10$ for $x \geq -4$.
Find the range of f.

$f(x) = (x + 4)^2 - 6$, with minimum at $x = -4$
$x = -4$ is in the domain, so the minimum value is -6.
Answer: $f(x) \geq -6$

2 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A model boat moves so that its position vector relative to O at time t s is $\mathbf{r} = (2t^2 + 4t)\mathbf{i} + (t^3 + 3t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (4t + 4)\mathbf{i} + (3t^2 + 3)\mathbf{j}$
$t = 1$: $\mathbf{v} = (8)\mathbf{i} + (6)\mathbf{j}$
Speed $= \sqrt{(8^2 + 6^2)}$
Answer: $\mathbf{v} = (8)\mathbf{i} + (6)\mathbf{j}$ m s^{-1}; speed 10 m s^{-1}

3 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.4. In a sample of 20 students, 15 pass first time.
Test, at the 5% significance level, whether the probability is different from 0.4.

$H_0: p = 0.4$, $H_1: p \neq 0.4$. Under H_0, $X \sim B(20, 0.4)$
$P(X \geq 15) = 1 - P(X \leq 14) = 0.0016$
Two-tailed: compare with 0.025. $0.0016 < 0.025$, so reject H_0
There is sufficient evidence that the probability is different from 0.4
Answer: reject H_0: sufficient evidence that the probability is different from 0.4

4 Fixed-point iteration (All boards)

(a) Show that $x^3 - 2x - 3 = 0$ can be written as $x = \sqrt[3]{2x + 3}$.
(b) Use $x_{n+1} = \sqrt[3]{2x_n + 3}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^3 = 2x + 3$, so $x = \sqrt[3]{2x + 3}$
(b) $x_1 = 1.9129$
$x_2 = 1.8969$
$x_3 = 1.8940$
Answer: $x_1 = 1.9129$, $x_2 = 1.8969$, $x_3 = 1.8940$

5 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t + 2$, $y = t^2 + 2t + 4$.
The line $y = x + 8$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 + 2t + 4 = 3t + 10$
$t^2 - t - 6 = 0$, so $(t - 3)(t + 2) = 0$
$t = -2$ or $t = 3$, giving the points $(-4, 4)$ and $(11, 19)$
Answer: $(-4, 4)$ and $(11, 19)$

6 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 40 and s.d. 6.
A random sample of 16 is taken. Find the probability that the mean time is less than 37.75 (3 s.f.).

$\overline{X} \sim N(40, \frac{6^2}{16})$, standard deviation 1.500
$z = \frac{37.75 - 40}{1.500} = -1.500$
$P = 0.0668$
Answer: 0.0668

7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 1 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{5}{12}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{4}{5} = T_2 \times \frac{12}{13}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 9.8$

From the first: $T_1 = \frac{15}{13} T_2$; into the second: $T_2 \times \frac{14}{13} = 9.8$

Answer: $T_1 = 10.5$ N (at α); $T_2 = 9.1$ N (at β)

8 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (6e^{4x} + \frac{2}{x})\,dx$.

$\int (6e^{4x} + \frac{2}{x})\,dx = \frac{3e^{4x}}{2} + 2\ln x$

$= [\frac{3e^{4x}}{2} + 2\ln x]_1^2$

$= \ln 4 + \frac{3e^8}{2} - \frac{3e^4}{2}$

Answer: $\ln 4 + \frac{3e^8}{2} - \frac{3e^4}{2}$

9 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 20 and common ratio 0.75.
Find the smallest n for which the nth term is less than 0.1.

$20 \times 0.75^{n-1} < 0.1$
$(n - 1)\lg 0.75 < \lg(0.1 \div 20)$
$\lg 0.75$ is negative, so dividing reverses the inequality:
$n - 1 > \lg(0.1 \div 20) \div \lg 0.75 = 18.417$, so $n > 19.417$
The smallest integer n is 20
Answer: $n = 20$

10 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x + 3\sec x - 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x + 3\sec x - 9 = 0$

$(\sec x + 3)(2\sec x - 3) = 0$, so $\sec x = -3$ or $\frac{3}{2}$

$\sec x = -3$, so $\cos x = -\frac{1}{3}$: $x = 109.5°, 250.5°$

$\sec x = \frac{3}{2}$, so $\cos x = \frac{2}{3}$: $x = 48.2°, 311.8°$

Answer: $x = 48.2°, 109.5°, 250.5°, 311.8°$