Day 91 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -7 \\ -4 \\ 4 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{7^2 + 4^2 + 4^2} = \sqrt{81} = 9$

Unit vector $= \frac{1}{9}\mathbf{a}$

Answer: $|\mathbf{a}| = 9$, unit vector $\frac{1}{9}\begin{pmatrix} -7 \\ -4 \\ 4 \end{pmatrix}$

2 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-x - 5| > -3x - 1$.

Critical values: $-x - 5 = -3x - 1$ gives $x = 2$; $-x - 5 = -(-3x - 1)$ gives $x = -\frac{3}{2}$

Sketch $y = |-x - 5|$ and $y = -3x - 1$, and compare the graphs at $x = 2$ and $x = -\frac{3}{2}$.

$|-x - 5| > -3x - 1$ when $x > -\frac{3}{2}$

Answer: $x > -\frac{3}{2}$

3 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 44.1 cm^2 and angle 1.8 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 44.1$, so $r^2 = \frac{2 \times 44.1}{1.8} = 49$, $r = 7$

Perimeter $= 2r + r\theta = 14 + 12.6 = 26.6$ cm

Answer: (a) 7 cm (b) 26.6 cm

4 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x + 1$ and $g(x) = x^2 + 2$ for $x \in \mathbb{R}$.
(a) Find $fg(x)$.
(b) Solve $gf(x) = 11$.

$fg(x) = f(x^2 + 2) = 2(x^2 + 2) + 1 = 2x^2 + 5$

$gf(x) = (2x + 1)^2 + 2 = 11$

$(2x + 1)^2 = 9$, so $2x + 1 = \pm 3$

$x = -2$ or $x = 1$

Answer: (a) $fg(x) = 2x^2 + 5$ (b) $x = -2$ or $x = 1$

5 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 9 kg and 12 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.
Take $g = 9.8$ m s^{-2}.
Find the acceleration and the tension in the string.

Heavier: $12g - T = 12a$

Lighter: $T - 9g = 9a$

Add: $3g = 21a$, so $a = 1.4$

$T = 9(9.8 + 1.4)$

Answer: $a = 1.4$ m s^{-2}; $T = 100.8$ N

6 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^2$ for $0 \le x \le 6$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 2). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^3/3]$ from 0 to 6 $= k \times \frac{6^3}{3} = 1$, so $k = \frac{1}{72}$

(b) $P(X > 2) = \int_2^6 \frac{1}{72} x^2\,dx = 1 - \frac{2^3}{6^3} = \frac{26}{27}$

(c) $E(X) = \int_0^6 x\,f(x)\,dx = \frac{1}{72} \times \frac{6^4}{4} = \frac{9}{2}$

Answer: (a) $k = \frac{1}{72}$ (b) $\frac{26}{27}$ (c) $\frac{9}{2}$

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $21\cos\theta + 20\sin\theta \equiv 29\cos(\theta - 43.6°)$,
solve $21\cos\theta + 20\sin\theta = -4$ for $0 \le \theta \le 360°$. (1 d.p.)

$29\cos(\theta - 43.6°) = -4$, so $\cos(\theta - 43.6°) = \frac{-4}{29}$

$\theta - 43.6° = \pm 97.9°$
$\theta = 43.6° + 97.9° = 141.5°$
$\theta = 43.6° - 97.9° = -54.3°$, so $\theta = 305.7°$
Answer: θ = 141.5°, 305.7°

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x - 4y = k$ is a tangent to the circle
$(x - 4)^2 + (y + 1)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(3, -4)$.
A unit step in this direction is $(3, -4)/5$, so the points of contact are
$(4, -1) \pm 10 \times (3, -4)/5 = (4, -1) \pm (6, -8)$
T = $(10, -9)$ or $(-2, 7)$, and $k = 3x - 4y$ gives:
k = 66 or k = −34
Answer: k = 66 or k = −34

9 Finding the mean or standard deviation (All boards)

The length X cm of a metal rod is normal with s.d. 10.
Given P(X < 68.4) = 0.8, find the mean (3 s.f.).

P(X < 68.4) = 0.8, so z = 0.8416
$\frac{68.4 - \mu}{10} = 0.8416$
$\mu = 68.4 - 0.8416 \times 10 = 60.0$
Answer: μ = 60.0

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{5}{12}$.

Take $g = 9.8\ m\ s^{-2}$.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{3}{5} = T_2 \times \frac{12}{13}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 58.8$

From the first: $T_1 = \frac{20}{13} T_2$; into the second: $T_2 \times \frac{21}{13} = 58.8$

Answer: T_1 = 56 N (at α); T_2 = 36.4 N (at β)