Day 90 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Inequalities with exponents and logs (All boards)

Solve $3^x < 323$, giving your answer to 3 s.f.

Take logs: $x \ln 3 < \ln 323$

$x < \dfrac{\ln 323}{\ln 3} = 5.259$

Answer: $x < 5.26$

2 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 16 m s^{-1} brakes and comes to rest with constant deceleration 5 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 16^2 - 2 \times 5 \times s$
$s = 256 \div 10 = 25.6$
(b) $0 = 16 - 5t$, so $t = 16 \div 5$
Answer: (a) 25.6 m (b) 3.2 s

3 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is (0, −2) and B is (2, −2).
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = (1, −2)
Radius$^2 = (0 - 1)^2 + (-2 - (-2))^2 = 1$
Answer: $(x - 1)^2 + (y + 2)^2 = 1$

4 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 30 kg is hinged at A and held horizontal by a light string attached at B.
The string makes an angle θ with the rod, where $\sin\theta = \frac{4}{5}$. Take g = 9.8 m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 3 = mg \times 1.5$
$T \times \frac{4}{5} \times 3 = 294 \times 1.5$
$T = 441 \div 2.4$
Answer: 183.75 N

5 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-x - 2| < 2x - 3$.

Critical values: $-x - 2 = 2x - 3$ gives $x = \frac{1}{3}$; $-x - 2 = -(2x - 3)$ gives $x = 5$

Sketch $y = |-x - 2|$ and $y = 2x - 3$, and compare the graphs at $x = \frac{1}{3}$ and $x = 5$.

$|-x - 2| < 2x - 3$ when $x > 5$
Answer: $x > 5$

6 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ax^n$. A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line
with intercept 0.778 and gradient 0.5.
Find a (2 s.f.) and n, then estimate y when x = 25 (3 s.f.).

$\log_{10} y = \log_{10} a + n \log_{10} x$
n = gradient = 0.5; $\log_{10} a = 0.778$, so $a = 10^{0.778} = 6.0$
$y = 5.998 \times 25^{0.5} = 30.0$
Answer: a = 6.0, n = 0.5, y = 30.0

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-3x-7}{(x+1)(x+3)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{-3x-7}{(x+1)(x+3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-3x-7}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$, so $-3x-7 \equiv A(x+3) + B(x+1)$
$x = -1$: $A = -2$; $x = -3$: $B = -1$

(b) $\int_0^2 \left(\frac{-2}{x+1} + \frac{-1}{x+3}\right) dx = [-2\ln(x+1) - \ln(x+3)]_0^2$
$= -2\ln\frac{3}{1} - \ln\frac{5}{3} = \ln\left(\frac{1}{15}\right)$

Answer: (a) $A = -2$, $B = -1$ (b) $\ln\left(\frac{1}{15}\right)$

8 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1-4x)^{-\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1+ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -\frac{3}{2}$ and $a = -4$:

coefficients: x: 6, x^2: 30, x^3: 140

Valid when $|4x| < 1$, that is $|x| < \frac{1}{4}$

Answer: (a) $1 + 6x + 30x^2 + 140x^3 + \ldots$ (b) $|x| < \frac{1}{4}$

9 Optimisation: open box and tank (All boards)

Squares of side x cm are cut from the corners of a square card of side 18 cm and the sides are folded up to make an open box.
Find x for the greatest volume, and that volume.

$V = x(18-2x)^2 = 4x^3 - 72x^2 + 324x$
$\frac{dV}{dx} = 12x^2 - 144x + 324 = (6x-18)(2x-18) = 0$
$x < 9$, so $x = 3$
$\frac{d^2V}{dx^2} = 24x - 144 = -72 < 0$ at $x = 3$, so maximum
$V = 3 \times 12^2 = 432$

Answer: $x = 3$, $V = 432$ cm^3

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
$P(X < 68) = 0.0668$ and $P(X > 88) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$68 = \mu + (-1.500)\sigma$ and $88 = \mu + 0.9998\sigma$
Subtract: $20 = 2.500\,\sigma$, so $\sigma = 8.00$
$\mu = 68 - (-1.500)(8.000) = 80.0$

Answer: $\mu = 80.0$, $\sigma = 8.00$