Day 9 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is -18 and the coefficient of x^2 is 135.
Find the values of n and k.

$nk = -18$, so $k = \frac{-18}{n}$

$\frac{n(n-1)}{2}k^2 = 135$, so $\frac{n(n-1)}{2} \times \frac{324}{n^2} = 135$

$\frac{(n-1) \times 324}{2n} = 135$, so $324(n-1) = 270n$

$n = 6$, and $k = -3$

Answer: $n = 6$, $k = -3$

2 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 + t^2 - 6t$ metres.
Find its velocity and acceleration when $t = 4$.

$v = ds/dt = 3t^2 + 2t - 6$

$a = dv/dt = 6t + 2$

$t = 4$: $v = 50$, $a = 26$

Answer: $v = 50$ m s^{-1}, $a = 26$ m s^{-2}

3 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{6}{\sqrt{6}} + \sqrt{96}$, giving your answer in the form $k\sqrt{6}$.

$\frac{6}{\sqrt{6}} = \frac{6\sqrt{6}}{6} = 1\sqrt{6}$

$\sqrt{96} = 4\sqrt{6}$

$1\sqrt{6} + 4\sqrt{6} = 5\sqrt{6}$

Answer: $5\sqrt{6}$

4 Finding the mean or standard deviation (All boards)

The mass X g of a flour bag is normal with mean 45.
Given $P(X < 47.5) = 0.95$, find the standard deviation (3 s.f.).

$z = 1.645$, so $\frac{47.5 - 45}{\sigma} = 1.645$

$\sigma = 2.5 \div 1.645 = 1.52$

Answer: $\sigma = 1.52$

5 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown horizontally at 7 m s^{-1} from the top of a platform 4.9 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $4.9 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 1$

$t = 1$ s

Horizontal: $x = 7 \times 1$

Answer: 1 s, 7 m

6 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $3x^2 + kx + 3 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$

$k^2 - 4 \times 3 \times 3 = 0$

$k^2 = 36$

$k = \pm 6$

Answer: k = ±6

7 Equations using sin²x + cos²x = 1 (All boards)

Solve $2 \sin x + 8 \cos^2 x - 5 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $2 \sin x + 3 - 8 \sin^2 x = 0$, i.e. $8 \sin^2 x - 2 \sin x - 3 = 0$

$(2 \sin x + 1)(4 \sin x - 3) = 0$, so $\sin x = -\frac{1}{2}$ or $\frac{3}{4}$

$\sin x = -\frac{1}{2}$: $x = 210.0°, 330.0°$

$\sin x = \frac{3}{4}$: $x = 48.6°, 131.4°$

Answer: x = 48.6°, 131.4°, 210.0°, 330.0°

8 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{3x^2}{64}$ for $0 \leq x \leq 4$, and 0 otherwise.

(a) Find $P(X > 1)$.
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^3}{64}$

(a) $P(X > 1) = 1 - F(1) = \frac{63}{64}$

(b) $F(m) = 0.5$: $\frac{m^3}{64} = \frac{1}{2}$

$m = 3.17$

Answer: (a) $\frac{63}{64}$ (b) 3.17

9 Inequalities with exponents and logs (All boards)

£1000 is invested at 10% compound interest per year.
Find the least whole number of years for the investment to exceed £5000.

$1000 \times 1.1^n > 5000$, so $1.1^n > 5$

$n > \frac{\ln 5}{\ln 1.1} = 16.89$

The least whole number is n = 17

Answer: 17 years

10 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 20°C: $\frac{dT}{dt} = -k(T - 20)$. Initially T = 90 and after 6 minutes T = 55.

Find k to 3 s.f. and the time to cool to 30°C, to 3 s.f.

$\frac{1}{T-20}\,dT = -k\,dt$, so $\ln(T-20) = -kt + c$, i.e. $T = 20 + 70e^{-kt}$

t = 6: $55 = 20 + 70e^{-6k}$, so $e^{-6k} = \frac{1}{2}$

$k = \frac{1}{6}\ln\frac{70}{35} = 0.1155$

$30 = 20 + 70e^{-kt}$: $t = \frac{1}{k}\ln\frac{70}{10} = 16.84$

Answer: k = 0.116, t = 16.8 minutes