**Day 89** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "If $x^2 > 81$ then $x > 9$."
Give a counter-example to show the statement is false.

$x^2 > 81$ is also true for negative x with $x < -9$.
Take $x = -10$: $x^2 = 100 > 81$, but $-10$ is not greater than 9.
**Answer: $x = -10$**

## 2 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2\cos t$, $y = 3\sin t$.
Find $\frac{dy}{dx}$ in terms of t, and its exact value when $t = \frac{\pi}{6}$.

$\frac{dx}{dt} = -2\sin t$, $\frac{dy}{dt} = 3\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{3\cot t}{2}$$

At $t = \frac{\pi}{6}$: $\frac{dy}{dx} = -\frac{3\sqrt{3}}{2}$

**Answer: $\frac{dy}{dx} = -\frac{3\cot t}{2}$; value $-\frac{3\sqrt{3}}{2}$**

## 3 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 3 kg hangs in equilibrium on a light string inclined at $\alpha$ to the vertical, held by a horizontal force P N, as shown. $\tan\alpha = \frac{3}{4}$.

Take $g = 9.8$ m s$^{-2}$.
Find the tension in the string and the value of P.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$
Vertical: $T\cos\alpha = mg = 29.4$, so $T = 29.4 \times 5/4$
Horizontal: $P = T\sin\alpha = 36.75 \times 3/5$
**Answer: T = 36.75 N; P = 22.05 N**

## 4 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = k\,u_n + 2$, $u_1 = 2$.
Given that $u_3 = 14$, find the possible values of k.

$u_2 = 2k + 2$
$u_3 = k(2k + 2) + 2 = 2k^2 + 2k + 2$
$2k^2 + 2k + 2 = 14$, so $2k^2 + 2k - 12 = 0$
$2(k - 2)(k + 3) = 0$
$k = -3$ or $k = 2$
**Answer: $k = -3$ or $k = 2$**

## 5 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 8 m and mass 20 kg hangs horizontally from two vertical strings attached 1.5 m from A and 1.5 m from B.

A particle of mass 4 kg is attached 2 m from A. Take $g = 9.8$ m s$^{-2}$. Find the tension in each string.

Moments about the string near A: $T_2(5) = 20\,g(2.5) + 4\,g(0.5)$
$T_2 = 101.92$
$T_1 + T_2 = 24\,g$ so $T_1 = 235.2 - 101.92$
**Answer: Near A: 133.28 N; near B: 101.92 N**

## 6 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $xe^y + 2y = -1$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(-1, 0)$.

Differentiate with respect to x: $x\frac{dy}{dx}e^y + 2\frac{dy}{dx} + e^y = 0$

$\frac{dy}{dx} = -\frac{e^y}{xe^y + 2}$

At $(-1, 0)$: gradient $= -1$

**Answer: $\frac{dy}{dx} = -\frac{e^y}{xe^y + 2}$; gradient $-1$**

## 7 Probability without replacement (All boards)

A bag has 4 green and 5 yellow beads; two are taken without replacement.

(a) Find P(exactly one green).

(b) Find P(at least one green).

(a) P(green then yellow) + P(yellow then green) $= 2 \times \frac{4}{9} \times \frac{5}{8} = \frac{5}{9}$

(b) $1 -$ P(both yellow) $= 1 - \frac{5}{9} \times \frac{4}{8} = 1 - \frac{5}{18} = \frac{13}{18}$

**Answer: (a) $\frac{5}{9}$ (b) $\frac{13}{18}$**

## 8 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t + 2$, $y = 2t^2 + 7t + 4$.

The line $y = x + 8$ meets the curve at two points.

Find the coordinates of these points.

Substitute x and y into the line: $2t^2 + 7t + 4 = 3t + 10$

$2t^2 + 4t - 6 = 0$, so $2(t - 1)(t + 3) = 0$

$t = -3$ or $t = 1$, giving the points $(-7, 1)$ and $(5, 13)$

**Answer: $(-7, 1)$ and $(5, 13)$**

## 9 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 4$. Use the Newton-Raphson method with $x_0 = 3$ to find $x_1$ and $x_2$, to 4 d.p.

$f'(x) = 1 + \frac{1}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{0.09861}{1.333} = 2.9260$

$x_2 = 2.9260 - \frac{-0.0003090}{1.342} = 2.9263$

**Answer: $x_1 = 2.9260$, $x_2 = 2.9263$**

## 10 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.

(a) Describe the correlation.

(b) Why should $y = 16 - x$ not be used for $x = 14$?

(a) The points lie loosely around a straight line falling: weak negative correlation

(b) The data cover $x = 2$ to 10; $x = 14$ is outside this range

so using the line would be extrapolation and may be unreliable

**Answer: (a) weak negative correlation (b) extrapolation outside the range of the data**