Day 88 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (All boards)

The mass X g of a flour bag is normal with mean 45, s.d. 8.
Find, to 3 s.f., the probability a value is
(a) more than 61,
(b) between 61 and 69.

$X \sim N(45, 8^2)$
(a) $P(X > 61) = 0.0228$
(b) $P(61 < X < 69) = 0.9987 - 0.9772 = 0.0214$
Answer: (a) 0.0228 (b) 0.0214

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x \leq 2x - 7$.

$-3x \leq -7$

Divide by -3 (reversing the inequality): $x \geq \frac{7}{3}$

Answer: $x \geq \frac{7}{3}$

3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of sec 300°,
giving your answer with a rational denominator.

$\sec 300° = \frac{1}{\cos 300°}$

$\cos 300° = \frac{1}{2}$, so $\sec 300° = \frac{1}{\frac{1}{2}}$

Answer: 2

4 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 3t^3 + 4t^2$ metres.
Find (a) the average velocity over the first 2 seconds, (b) the velocity at t = 2.

(a) $s(2) = 40$, so average velocity $= 40 \div 2$
(b) $v = ds/dt = 9t^2 + 8t$, $v(2) = 52$
Answer: (a) 20 m s^{-1} (b) 52 m s^{-1}

5 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^2 \frac{3}{3x + 1}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{3}{3x + 1}\,dx = \ln(3x + 1)$

$= [\ln(3x + 1)]_0^2$

$= \ln 7$

Answer: ln 7

6 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which

$3x + 5 > 5$ and $x^2 + x - 6 > 0$.

$3x + 5 > 5$ gives $x > 0$

$x^2 + x - 6 > 0$: $(x + 3)(x - 2) > 0$ gives $x < -3$ or $x > 2$

Both must hold: $x > 2$

Answer: $x > 2$

7 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 0.5) and (4, 2.1). Find A to 3 s.f. and k.

$\ln y = \ln A + kx$, so $\ln A = 0.5$ and $A = e^{0.5} = 1.649$

$k = \text{gradient} = \frac{2.1 - 0.5}{4 - 0} = 0.4$

Answer: A = 1.65, k = 0.4

8 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 - 2t + 2$ m s^{-1}. Find the distance moved between t = 2 and t = 5. (The particle does not change direction.)

$s = \int_2^5 (3t^2 - 2t + 2)\,dt$

$= [\,t^3 - t^2 + 2t\,]$ from t = 2 to t = 5

$= 110 - 8$

Answer: 102 m

9 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 60) = 0.0228 and P(X > 90) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$

$60 = \mu + (-1.999)\sigma$ and $90 = \mu + 0.9998\sigma$

Subtract: $30 = 2.999\,\sigma$, so $\sigma = 10.0$

$\mu = 60 - (-1.999)(10.00) = 80.0$

Answer: μ = 80.0, σ = 10.0

10 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Show that the equation $3x^2 - 8x + 10 = 0$ has no real roots.

$b^2 - 4ac = (-8)^2 - 4 \times 3 \times 10$

$= 64 - 120 = -56$

The discriminant is negative, so there are no real roots.

Answer: $b^2 - 4ac = -56 < 0$