Day 87 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 5e^x - 6 = 0$,
giving any answers as exact values.

Let $y = e^x$: $y^2 - 5y - 6 = 0$
$(y + 1)(y - 6) = 0$
$e^x > 0$, so $y = -1$ is rejected; $e^x = 6$
$x = \ln 6$
Answer: $x = \ln 6$

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration. Its initial speed is 3 m s^{-1}, its final speed is 23 m s^{-1} and it travels 52 m.
Find the time taken.

$s = \frac{1}{2}(u + v)t$
$52 = \frac{1}{2} \times (3 + 23) \times t$
$t = 104 \div 26$
Answer: 4 s

3 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of
$\int_0^{\frac{\pi}{2}} x \sin x \, dx$.

Let $u = x$ and $\frac{dv}{dx} = \sin x$, so $\frac{du}{dx} = 1$ and $v = -\cos x$

Integral $= [uv] - \int v\frac{du}{dx} \, dx$

$= [-x \cos x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos x \, dx$

$= 1$
Answer: 1

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 15 pairs of data, $S_{xx} = 150$, $S_{yy} = 150$ and $S_{xy} = -57$.
Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-57}{\sqrt{150 \times 150}}$

$= \frac{-57}{150.00} = -0.380$

r is negative, so there is weak negative correlation
Answer: $r = -0.380$, weak negative correlation

5 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a minimum point at (−4, 6).
Find the turning point on the curve:
(a) y = f(−x) (b) y = −f(x)
(c) $y = f(\frac{x}{4})$

(a) y = f(−x): the point (−4, 6) maps to (4, 6)
(b) y = −f(x): the point (−4, 6) maps to (−4, −6)
(c) $y = f(\frac{x}{4})$: the point (−4, 6) maps to (−16, 6)
Answer: (a) (4, 6) (b) (−4, −6) (c) (−16, 6)

6 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 20, is a bar 4 cm wide, 1.25 cm tall.
(a) Find the bar height for width 20, frequency 45.
(b) Find the frequency of a 1 cm by 3 cm bar.

Area of bar $\propto$ frequency: $4 \times 1.25 = 5$ cm^2 represents 20
so 1 unit of frequency = 0.25 cm^2
(a) Width = 4 cm, area = $0.25 \times 45 = 11.25$ cm^2, height = $11.25 \div 4 = 2.8125$ cm
(b) Area = $1 \times 3 = 3$ cm^2, frequency = $3 \div 0.25 = 12$
Answer: (a) 2.8125 cm (b) 12

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $24\cos\theta + 7\sin\theta \equiv 25\cos(\theta - 16.3°)$,
solve $24\cos\theta + 7\sin\theta = 4$ for $0 \le \theta \le 360°$. (1 d.p.)

$25\cos(\theta - 16.3°) = 4$, so $\cos(\theta - 16.3°) = \frac{4}{25}$
$\theta - 16.3° = \pm 80.8°$
$\theta = 16.3° + 80.8° = 97.1°$
$\theta = 16.3° - 80.8° = -64.5°$, so $\theta = 295.5°$
Answer: θ = 97.1°, 295.5°

8 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A javelin is projected from level ground with horizontal velocity component 23 m s^{-1} and vertical velocity component 39.2 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 39.2\,t - 4.9t^2$, so $t = 39.2 \div 4.9$
(b) Range = horizontal speed × time = 23×8
Answer: (a) 8 s (b) 184 m

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers
m and n such that $6m + 9n = 56$.

Assume there are integers m and n with $6m + 9n = 56$.
$6m + 9n = 3(2m + 3n)$.
So the left-hand side is a multiple of 3.
But $56 = 3 \times 18 + 2$ is not a multiple of 3.
This is a contradiction, so no such integers exist.
Answer: no integer solutions (LHS is a multiple of 3, 56 is not)

10 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "If $x^2 > 16$ then $x > 4$."
Give a counter-example to show the statement is false.

$x^2 > 16$ is also true for negative x with $x < -4$.
Take $x = -5$: $x^2 = 25 > 16$, but -5 is not greater than 4.
Answer: x = −5