Day 86 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + n + 5$ is odd for all integers n.

$n^2 + n = n(n + 1)$
As 1 is odd, n and n + 1 have opposite parity,
so one of them is even and the product n(n + 1) is even.
Even + 5 = odd, so $n^2 + n + 5$ is odd.
Answer: n(n + 1) is even, so the expression is odd

2 Normal distribution probabilities (All boards)

A journey takes X minutes, normal with mean 20, s.d. 4.
Find, to 3 s.f., the probability a value is
(a) less than 14,
(b) between 14 and 20.

$X \sim N(20, 4^2)$
(a) $P(X < 14) = 0.0668$
(b) $P(14 < X < 20) = 0.5000 - 0.06681 = 0.433$
Answer: (a) 0.0668 (b) 0.433

3 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form 9k + 5, where k is an integer.

Write the integer as n = 9q + j, where j = 0, 1, ..., 8.
n^3 leaves the same remainder as j^3 when divided by 9.
j = 0, 1, 2, 3, 4, 5, 6, 7, 8 give remainders 0, 1, 8, 0, 1, 8, 0, 1, 8.
The only possible remainders are 0, 1, 8, never 5.
Answer: possible remainders 0, 1, 8, never 5

4 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = kx^2$ for x = 1, 2, ..., 6.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(1 + 4 + 9 + 16 + 25 + 36) = 1$, so $91k = 1$ and $k = \frac{1}{91}$

(b) $E(X) = 1 \times \frac{1}{91} + 2 \times \frac{4}{91} + 3 \times \frac{9}{91} + 4 \times \frac{16}{91} + 5 \times \frac{25}{91} + 6 \times \frac{36}{91} = \frac{63}{13}$

$E(X^2) = 25$, so $Var(X) = 25 - (\frac{63}{13})^2 = \frac{256}{169}$

Answer: (a) $k = \frac{1}{91}$ (b) $E(X) = \frac{63}{13}$, $Var(X) = \frac{256}{169}$

5 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 4 + 4 \sin(\frac{\pi t}{12})$, where t is in hours after midnight.

Find (a) the greatest and least values, (b) the values of t with $0 \leq t \leq 24$ when it equals 2 m.

(a) sin ranges from −1 to 1: greatest 4 + 4 = 8, least 4 − 4 = 0

(b) $4 + 4 \sin(\frac{\pi t}{12}) = 2$, so $\sin(\frac{\pi t}{12}) = -\frac{1}{2}$

$\frac{\pi t}{12} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so t = 14 or 22

Answer: (a) 8 m, 0 m (b) t = 14 and 22

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 6?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(−2, −4) becomes (−2, 4)
(b) The maximum of the middle part is 4. Compare k = 6 with 4:
k > 4 gives 2 intersections with y = k.
Answer: (a) (−2, 4) (b) 2

7 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 60 kg stands on the floor of a lift. The lift accelerates upwards at 1.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the normal reaction between the person and the floor.

Resolve vertically (up positive): R − 60 × 9.8 = 60 × 1.5
R = 60(9.8 + 1.5)
Answer: 678 N

8 Optimisation: open box and tank (All boards)

An open tank has a square base of side x m and volume 32 m^3.
Show that its surface area is $S = x^2 + \frac{128}{x}$, and find the minimum value of S.

Height $h = \frac{32}{x^2}$, so $S = x^2 + 4xh = x^2 + \frac{128}{x}$

$\frac{dS}{dx} = 2x - \frac{128}{x^2} = 0$, so $x^3 = 64$ and x = 4

$\frac{d^2S}{dx^2} = 2 + \frac{256}{x^3} > 0$, so minimum

$S = 16 + 32 = 48\ m^2$
Answer: S = 48 m^2 (when x = 4)

9 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 60 kW. The resistance to motion is constant at 500 N.
Find the acceleration when the speed is 8 m s^{-1}.

P = Fv: F = 60000 ÷ 8 = 7500 N
F − R = ma: 7500 − 500 = 1000 a
Answer: 7 m s^{-2}

10 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 0.5) and (5, 3.5).
Find A to 3 s.f. and k.

ln y = ln A + kx, so ln A = 0.5 and $A = e^{0.5} = 1.649$

k = gradient = $\frac{3.5 - 0.5}{5 - 0} = 0.6$

Answer: A = 1.65, k = 0.6