Day 85 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

7 values of x are coded using $y = \frac{(x - 100)}{5}$.

$\Sigma y = -6$ and $\Sigma y^2 = 150$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = -6 \div 7 = -0.8571$

Mean of $x = 100 + 5 \times -0.8571 = 95.7$

Variance of $y = \frac{150}{7} - (-0.8571)^2 = 20.69$

SD of y = 4.549, so SD of $x = 5 \times 4.549 = 22.7$

Answer: mean 95.7, standard deviation 22.7

2 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:

(a) y = −f(x)

(b) y = f(2x)

(c) y = 3f(x)

(a) y = −f(x) is a reflection in the x-axis.

(b) y = f(2x) is a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.

(c) y = 3f(x) is a stretch, scale factor 3, parallel to the y-axis.

Answer: (a) reflection in the x-axis (b) stretch, scale factor $\frac{1}{2}$, parallel to the x-axis (c) stretch, scale factor 3, parallel to the y

3 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 11 \\ -3 \\ -7 \end{pmatrix}$.

Find AB and the distance AB.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \begin{pmatrix} 12 \\ -4 \\ -3 \end{pmatrix}$

$|\mathrm{AB}| = \sqrt{12^2 + 4^2 + 3^2} = \sqrt{169} = 13$

Answer: AB = $\begin{pmatrix} 12 \\ -4 \\ -3 \end{pmatrix}$, distance 13

4 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 10 kg is pulled along rough horizontal ground by a horizontal force of 44.2 N. It accelerates at 0.5 m s^{-2}.

Take g = 9.8 m s^{-2}.

Find the coefficient of friction between the sledge and the ground.

$R = mg = 10 \times 9.8 = 98$

Resultant: $44.2 - F = 10 \times 0.5$, so $F = 39.2$

$F = \mu R$: $\mu = 39.2 \div 98$

Answer: 0.4

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when

$2x^3 + 3x^2 - 2x - 6$ is divided by $(x + 2)$.

$2x^2 \times (x + 2) = 2x^3 + 4x^2$, leaving $-x^2 - 2x - 6$

$-x \times (x + 2) = -x^2 - 2x$, leaving -6

Quotient $2x^2 - x$, remainder -6

Answer: quotient $2x^2 - x$, remainder -6

6 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $(-2\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(4\mathbf{i} + \mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-2\mathbf{i} + 6\mathbf{j}) + \mathbf{P} = 5(4\mathbf{i} + \mathbf{j}) = (20\mathbf{i} + 5\mathbf{j})$

$\mathbf{P} = 22\mathbf{i} - \mathbf{j}$

$|\mathbf{P}| = \sqrt{(22^2 + -1^2)}$

Answer: $\mathbf{P} = (22\mathbf{i} - \mathbf{j})$ N; $|\mathbf{P}| = 22.0$ N

7 Equations using sin²x + cos²x = 1 (All boards)

Solve $5 \sin x + 3 \cos^2 x - 5 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $5 \sin x - 3 \sin^2 x - 2 = 0$, i.e. $3 \sin^2 x + 2 - 5 \sin x = 0$

$(3 \sin x - 2)(\sin x - 1) = 0$, so $\sin x = \frac{2}{3}$ or 1

$\sin x = \frac{2}{3}$: $x = 41.8°, 138.2°$

$\sin x = 1$: $x = 90.0°$

Answer: $x = 41.8°, 90.0°, 138.2°$

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 5 ml. Set mean 100 ml; 36 bottles have mean 101.5 ml.
Test at the 5% level whether the true mean differs from 100.

$H_0: \mu = 100$, $H_1: \mu \neq 100$. Under H_0, $\overline{X} \sim N(100, \frac{5^2}{36})$

$z = \frac{101.5 - 100}{5/\sqrt{36}} = 1.800$

$P(Z > 1.800) = 0.0359 > 0.025$

Do not reject H_0: there is insufficient evidence that the mean has changed

Answer: do not reject H_0: insufficient evidence of a change in the mean

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-3x^2 + 12x - 30}{(x - 5)(x - 2)^2}$ in partial fractions.

$$\frac{-3x^2 + 12x - 30}{(x - 5)(x - 2)^2} \equiv \frac{A}{(x - 5)} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$$

$-3x^2 + 12x - 30 \equiv A(x - 2)^2 + B(x - 5)(x - 2) + C(x - 5)$

$x = 5$ gives $A = -5$; $x = 2$ gives $C = 6$

Coefficients of x^2: $-3 = A + B$, so $B = 2$

Answer: $-\frac{5}{(x - 5)} + \frac{2}{(x - 2)} + \frac{6}{(x - 2)^2}$

10 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 12 + 2 \sin(\frac{\pi t}{6})$, where t is in hours after noon.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 12$ when it equals 13 m.

(a) sin ranges from -1 to 1: greatest $12 + 2 = 14$, least $12 - 2 = 10$

(b) $12 + 2 \sin(\frac{\pi t}{6}) = 13$, so $\sin(\frac{\pi t}{6}) = \frac{1}{2}$

$\frac{\pi t}{6} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so $t = 1$ or 5

Answer: (a) 14 m, 10 m (b) t = 1 and 5