Day 84 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(\mathbf{i} + \mathbf{j})$ m s^{-2}.
Find its velocity and speed when t = 6.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-\mathbf{i} + 6\mathbf{j}) + 6(\mathbf{i} + \mathbf{j})$
$= 5\mathbf{i} + 12\mathbf{j}$
Speed $= \sqrt{(5^2 + 12^2)}$
Answer: $\mathbf{v} = (5\mathbf{i} + 12\mathbf{j})$ m s^{-1}; speed 13 m s^{-1}

2 Binomial expected frequency (All boards)

A biased coin with P(head) = $\frac{1}{6}$ is tossed 8 times; X is the number of heads.

This is done 200 times. Estimate the number of times that X = 2.

$X \sim B(8, \frac{1}{6})$

$P(X = 2) = {}^8C_2 \times (\frac{1}{6})^2 \times (\frac{5}{6})^6 = 0.2605$

Expected $= 200 \times 0.2605 = 52.10$
Answer: about 52 times

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 6 cm and angle 300°.
Find the perimeter of the sector, in terms of π.

$\theta = \frac{5\pi}{3}$ radians

Arc length $= r\theta = 10\pi$ cm
Perimeter = 2r + arc $= 12 + 10\pi$ cm
Answer: $12 + 10\pi$ cm

4 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 6x - 4y + 5 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:
$(x - 3)^2 - 9 + (y - 2)^2 - 4 + 5 = 0$
$(x - 3)^2 + (y - 2)^2 = 8$
Centre (3, 2), radius $2\sqrt{2}$
Answer: centre (3, 2), radius $2\sqrt{2}$

5 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 2 : 3, and N lies on OB with ON : NB = 3 : 2.
Find OM and NM in terms of a and b.

$\mathbf{AB} = \mathbf{b} - \mathbf{a}$, so $\mathbf{AM} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$

$\mathbf{OM} = \mathbf{a} + \mathbf{AM} = \frac{3\mathbf{a}}{5} + \frac{2\mathbf{b}}{5}$

$\mathbf{ON} = \frac{3\mathbf{b}}{5}$, so $\mathbf{NM} = \mathbf{OM} - \mathbf{ON} = \frac{3\mathbf{a}}{5} - \frac{\mathbf{b}}{5}$

Answer: $\mathbf{OM} = \frac{3\mathbf{a}}{5} + \frac{2\mathbf{b}}{5}$, $\mathbf{NM} = \frac{3\mathbf{a}}{5} - \frac{\mathbf{b}}{5}$

6 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} - 10\times2^{x} + 16 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 - 10y + 16 = 0$

$(y - 8)(y - 2) = 0$

$y = 8$ or $y = 2$

$2^x = 8$ gives $x = 3$; $2^x = 2$ gives $x = 1$.

Answer: $x = 1$ or $x = 3$

7 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards at 14.7 m s^{-1} from a point 49 m above horizontal ground.

Take g = 9.8 m s^{-2}.

Find the time before it hits the ground.

Taking up as positive: $-49 = 14.7\,t - 4.9t^2$

$4.9t^2 - 14.7\,t - 49 = 0$

$t = (14.7 \pm \sqrt{1176.49}) \div 9.8 = (14.7 \pm 34.3) \div 9.8$

Positive root only.

Answer: 5 s

8 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.1, find the least number of tickets needed for the probability of at least one success to exceed 0.95.

P(at least one) $= 1 - 0.9^n > 0.95$

$0.9^n < 0.05$

$n > \dfrac{\log 0.05}{\log 0.9} = 28.43$

Answer: $n = 29$

9 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{4}} x \sin 2x \, dx$.

Let $u = x$ and $\dfrac{dv}{dx} = \sin 2x$, so $\dfrac{du}{dx} = 1$ and $v = -\dfrac{\cos 2x}{2}$

Integral $= [uv] - \int v\dfrac{du}{dx}\,dx$

$= \left[-\dfrac{x\cos 2x}{2}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} -\dfrac{\cos 2x}{2}\,dx$

$= \dfrac{1}{4}$

Answer: $\dfrac{1}{4}$

10 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (−5, 5) and (2, 7).

(a) Find the midpoint of AB.

(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint $= \left(\dfrac{-5+2}{2}, \dfrac{5+7}{2}\right) = \left(-\dfrac{3}{2}, 6\right)$

(b) Gradient $= \dfrac{2}{7}$

$y - 5 = \dfrac{2}{7}(x + 5)$

Multiply out and collect terms: $2x - 7y + 45 = 0$

Answer: (a) $\left(-\dfrac{3}{2}, 6\right)$ (b) $2x - 7y + 45 = 0$